A

A CONCISE but COMPREHENSIVE

TREATISE

Of VULGAR and DECIMAL

ARITHMETIC:

Together with

A COMPENDIUM of ALGEBRA.

Wherein the RUDIMENTS of that admired ART are made easy.

To which is added,

A plain and familiar Investigation and Illustration of the THEOREMS in Simple and Compound INTEREST, ANNUITIES, purchasing of FREEHOLD ESTATES, ANNUITIES ON LIVES, &c. The whole being conducted in such a Manner as to render it generally useful.

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LONDON:
Printed for W. GOLDSMITH, No. 24, Pater-noster Row, London.
MDCCLXXIII.

CONTENTS.

ERRATA.

Page 53, line 2, for *he* read *ſhe.*—p. 60, l. laſt, for *than there will* read *then will.*—P. 62, l. 20 and 21, for 19 *oz.* read 14 *oz.*—P. 64, for $61\frac{1}{7}$ *anſwer* read $61\frac{5}{7}$ *anſwer.*—P. 83, for $4\frac{18}{19}$ *anſwer* read $4 = \frac{18}{9}$ *anſwer.*—P. 140, l. 1, for *by* read *be.*—P. 146, l. 10, for $602 = x$ read $600 = x$.—P. 159, for *Art.* 172 read *Art.* 169.—P. 175, for *t* = *t the time required* read 7 = *t the time required.*—P. 177, for *the log. of* $p = 610{\cdot}25$ read *the log. of* $p = 600{\cdot}25$.—P. 183, l. 12, for *Art.* 192, read *Art.* 190.

PREFACE.

THAT the ſcience of Arithmetic is capable of much improvement is beyond a doubt, and to the many excellent books on the ſubject it is that we are indebted for the advantages it has already received ; every attempt to render it more compleat ought to merit our encouragement, and for this very reaſon it is that I have ventured to publiſh this Treatiſe. Many books on the ſubject are ſo voluminous (and add to that expenſive) that they deter a common reader from an attentive peruſal ; others again are no more than ſuperficial abſtracts, ſo that there can be little ſolid arithmetical knowledge gained from them. If this treatiſe has any merit, it will ariſe from my endeavouring to avoid both extreams. In attempting it, I have proceeded from article to article, with occaſional references, by which much of that tautology, found in voluminous books of the kind, is avoided, and this alſo allows me to be ſufficiently explicit upon more material and inſtructive parts of the ſcience. The demonſtrations and reaſons for the ſeveral rules are generally in notes at the bottom of the page, and in a ſmaller type, which ſtill helps to keep the book in leſs compaſs, and at leſs expence : in ſhort, I have taken every means in my power to compile the whole for the benefit of my readers, not by expanding ſome parts and curtailing others (a fault too common in many who have writ upon the ſubject) but have ſtudied to ex-

preſs myſelf in as eaſy and familiar a manner as poſſible. The directions and rules are, I hope, plain and conciſe, and free from that ambiguity which abounds in prolix and trivial diſtinctions, which ſeldom fail to explain the meaning away. In purſuance of my plan, I begin with Notation, and endeavour to make this firſt part of arithmetic as familiar as I can, becauſe in teaching it is too much neglected: I have converſed with ſeveral, who thought themſelves maſters of arithmetic, yet could read no more than nine figures; becauſe, ſay they, our numeration table goes to no greater length: but how ignorant ſuch are of the true method of notation is apparent to any who underſtands the nature of numbers. In the next place, the common abbreviations and arithmetical characters are explained, which is of more uſe in accompts than many imagine; they greatly contract and beautify the work, and, when rightly underſtood, render it more eaſy; for a ſolution expreſſed with proper characters may generally be comprehended or taken in by the eye at one view, and conſequently more readily, and with greater force conveyed to the underſtanding. The four fundamental rules, *viz.* ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION, I treat of in integral numbers, and then proceed to go over them again in the different denominations of money, weights, &c. which is certainly the natural order of the rules; and I am convinced, that it is the readieſt way to inform and inſtruct the ſtudent in the principles of the ſcience; for how can any ſuppoſe, that a beginner will reaſon and inform himſelf of the nature of Compound

pound Addition, when he is ignorant of Division of whole numbers? which is the only rule we can make use of to determine the numbers to be carried and set down, especially where it takes a great number of the inferior denomination to make one of the next superior. Reduction is the next rule in the order of this book; in avoirdupois Weight I have illustrated the method of reducing hundred weights to pounds in a variety of contracted methods, because it occurs in the practice of several mercantile affairs. I have next introduced a sketch upon the ratio, or proportion of numbers, which I designed as an introduction to the Rules of Three, commonly distinguished by the appellations of Golden Rule Direct, and Golden Rule Inverse, or Indirect, which follow next in order. And thus far I would recommend the same order to be observed in teaching as is followed in this treatise; though I would not be understood in so confined a sense as to allow nothing for the abilities, circumstances, time, &c. of the learner: It many times happens that the learner cannot attend more than three or four weeks, and in such like cases the teacher may be allowed to transgress the most connected rules of science, and to give the scholar those hints which are best adapted to his business, abilities, and time. As to the order of the rules commonly taught after the Golden Rule, 'tis of little consequence. In my method, the rule of Compound Proportion is next the Rule of Three Inverse; it extends to any proportion whatsoever, whether direct or inverse, and to any number of terms. The method of solution and setting down the work I would strongly

recommend for its beauty, concisenefs, and (when understood) even for its simplicity; for if the work will admit of contracting, you immediately can find out the numbers, and know by inspection what quantities may be expunged, &c. The methods I have laid down in Practice will be found different from those of several authors, and, I presume, new to several of my readers, particularly the fourth rule, which directs to take the parts from two shillings, but by this means you always evade the trouble of reducing the answer in shillings to pounds, and in a great many questions may perform the work with a third part of the figures. I cannot say too much to induce the ingenious learner to the study of Vulgar Fractions; for few, very few questions in arithmetic can be proposed which may not be greatly contracted in some part or other by a thorough knowledge of fractions. I have, in the solutions to the questions, followed the same method in placing down the work as in the rule of Compound Proportion, by which it will appear that most of the operations are much abbreviated. In Decimal Arithmetic I have explained the nature and use of infinite or circulating decimals, which I take to be very necessary, as several common questions in arithmetic, by the ordinary decimal operation, would not come out to any thing like a true conclusion, and yet by the method of circulates the answer will be found exact, and shorter also. After the doctrine of Vulgar and Decimal Fractions, I have introduced several rules in arithmetic (which were purposely omitted) as proper exercises to the said principles; and after these, I proceed

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to Algebra, where I am as explicit in the rudiments as poſſible, and have not omitted the explanation of any of the principals of this univerſal arithmetic which may contribute to the advantage of the learner in peruſing a large tract, which is the deſign of this compendium. I have thought proper to reſerve the extraction of the Square and Cube Root till after the Evolution of Algebraic Quantites; my reaſon for it was this, that many arithmeticians knowing no more of the extraction of Roots but what they have received from the rule, as ſoon as that is forgotten, their knowledge of the matter is likewiſe at an end; whereas he who derives his knowledge from Algebra, will be able (though he has forgot his rule) to make formulæ or rules of his own by which he can proceed; and for the ſame reaſon the reader will find the progreſſions, following thoſe practical queſtions in Algebra, which produce ſimple equations. After a few practical queſtions are propoſed and ſolved, in order to illuſtrate the method of reducing Quadratic Equations, I preſume, the reader will be qualified to enter upon Intereſt and Annuities, which are next in order, the ſtudy of which will, I imagine, require no other recommendation than the extenſive and frequent uſe we find them of· there are few people but are ſome way or other connected in buying or ſelling annuities, widow rights, penſions, reverſions of eſtates, &c. the methods of computing theſe are illuſtrated with proper examples, and ſeveral uſeful and curious queſtions added to the end of Annuities on Lives to explain the extenſiveneſs and utility of computations of this kind:—And that nothing may

may be wanting to make the book as compleat as is in my power, I have concluded the whole with a collection of practical Compendiums and Bills of Parcels, adapted to the moſt common affairs relating to the trading and buſy part of mankind. I hope the candid reader will overlook any defects he ſhall find in the enſuing ſheets; I am very far from imagining the work to be perfect, but I hope, on the other hand, it will be found of uſe to the young tyro, which was my deſign when I firſt reſolved to publiſh the book. There are ſeveral teachers and others who make an objection to teaching by a book; becauſe, ſay they, notwithſtanding any care which can be taken to prevent them, boys will be looking into the manuſcripts of their ſchool-fellows, and the ſame courſe of examples ſo often repeated will certainly become familiar in any ſchool. The truth of this is but too obvious, and for that reaſon I would recommend a ſtudied variation of the queſtions propoſed, eſpecially in the numerical part, and not allow (which is too often the final determination) the agreement or diſagreement of the ſcholar's anſwer with the author or teacher's; but cauſe the pupil to prove his proceſs either by varying the conditions of the given queſtion, or elſe let him perform the ſame queſtions by different methods or rules, for which reaſon I have not given ſo many queſtions under many of the rules as ſome might expect, nor are they always new, for I do not imagine that the writing of a new book upon arithmetic conſiſts in a ſet of new queſtions, which are the ſame in ſenſe as hundreds that have been publiſhed before; but as it

were

were in endeavouring to new-model or amend the methods of expreſſion and ſolution, and in proceeding in a more connected and conciſe manner. And as I would not have the ſcholar to be entirely dependent on any author for his arithmetical knowledge, neither would I have him to fall into the much worſe extream of never at all conſulting any. Though I have known teachers forbid their ſcholars to dip into any books upon the ſubject, for fear of ſpoiling them; but how preſuming ſuch Gentlemen are of their own incomprehenſible abilities is too groſs to need any remark. I think it may as well be argued that the beſt and readieſt way to attain the knowledge of architecture, is to blindfold the eyes of the artiſt when he is about to draw the plan of ſome notable building.

I return my worthy Subſcribers ſincere Thanks for their encouragement, and hope the book will, in general, pleaſe them; which is the moſt hearty wiſh of

Their humble ſervant,

W. THOMPSON.

were in endeavouring to answer their [illegible] the methods of expression and [illegible] and in proceeding in a more connected and concise manner; and as I would not have the scholar to be [illegible] dependent on any author for his arithmetical knowledge, neither would I have him to fall into the [illegible] at rote all calculating ways. There are some known teachers that forbid their scholars to dip into any book upon the subject, for fear of spoiling them: but how pretty is such [illegible] gentlemen are of their [illegible] the want of reading any [illegible] I think it may [illegible] that the best and readiest way to attain the knowledge of architecture, is to blindfold the eyes of the artist when he is about to draw the plan of some noble building.

I return my worthy Subscribers sincere Thanks for their encouragement, and hope this book will, in general, please them; which is the most hearty wish of

Their humble Servant,

W. THOMSON.

A

TREATISE

Of Vulgar and Decimal

ARITHMETIC, &c.

A concise, but comprehensive Treatise of Vulgar and Decimal Arithmetic, together with a Compendium of Algebra, Annuities, Valuation of Annuities on Lives, &c.

Definition.

1. ARITHMETIC is that science which describes the properties and power of numbers, and by them deduces precepts of computation, which are necessary, not only in common occurrences relative to the busy part of mankind, but likewise in assisting the speculations of the curious in the higher arts and sciencies.

Notation.

2. This is the first thing to be considered in arithmetic, and shews how to express any numerical quantity by certain characters called *figures* or *digits*, of these there are but ten in use, *viz.*

1	One
2	Two
3	Three
4	Four
5	Five
6	Six
7	Seven
8	Eight
9	Nine
0	Cypher or nothing

Any quantity above nine, requires more of theſe digits than one to expreſs the number of units or ones which are contained therein, as ſpecified in the following table.

1	One or an unit
10	Ten
100	One hundred
1000	One thouſand
10000	Ten thouſand
100000	One hundred thouſand
1000000	One million.

3. It muſt be obſerved, that by removing the unit one place more towards the left hand, and annexing a cypher thereto, it becomes ten units; with two cyphers placed after it, one hundred units; the affixing of every nothing or cypher, cauſing an increaſe of the value in a tenfold proportion or ratio, which is the general property of theſe digits, in compoſing any number whatever.

4. When a quantity does not conſiſt of an exact number of tens, then are the places of theſe cyphers ſupplied with other ſignificant and proper digits: Thus 14 is read fourteen, 16 reads ſixteen; alſo two tens or twenty is expreſſed with 20, thirty with 30, &c. the intermediate terms being ſupplied with proper digits in the units place, as twenty-ſix is 26, 48 forty-eight, &c.

5. Hence we obtain the method of reading and writing any numerical quantity, as exhibited in the following table.

NU-

NUMERATION TABLE.

Millions of millions or billions	Hundreds of thousands of millions	Tens of thousands of millions	Thousands of millions	Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units	
												1	one
											2	1	twenty-one
										3	2	1	three hundred and twenty-one
									4	3	2	1	4 thousand 321
								5	4	3	2	1	54 thousand 321
							6	5	4	3	2	1	654 thousand 321
						7	6	5	4	3	2	1	7 million 654 thousand 321
					8	7	6	5	4	3	2	1	87 million 654 thousand 321
				9	8	7	6	5	4	3	2	1	987 million 654 thousand 321
			1	9	8	7	6	5	4	3	2	1	1 thousand 987 million 654 thousand 321
		2	1	9	8	7	6	5	4	3	2	1	21 thousand 987 million 654 thousand 321
	3	2	1	9	8	7	6	5	4	3	2	1	321 thousand 987 million 654 thousand 321
4	3	2	1	9	8	7	6	5	4	3	2	1	4 million of millions (or four billions), &c.

6. Now it is evident, that to expreſs the value of any quantity in words, we muſt name the digit and the place it ſtands in, as ſpecified by the table. For inſtance, 7609546 is read 7 million 6 hundred and 9 thouſand, 5 hundred and forty-ſix.

7. It may be obſerved, that the ſeventh place is millions, the thirteenth millions of millions, or bil-

 lions

lions, the nineteenth, millions of millions, or trillions, &c. therefore, where there are many figures, they are very aptly divided into periods of ſix figures each, and thoſe again ſubdivided into thoſe of three, thus:

Hundreds of quatrillions	Hundreds of thouſands of trillions	Hundreds of trillions	Hundreds of thouſands of billions	Hundreds of billions	Hundreds of thouſands of millions	Hundreds of millions	Hundreds of thouſands	Hundreds
745	876	543	987	496	583	978	476	537
.		.			.	.		
.		.			.			
.		.						
.								

The uſe of theſe points below the line, is to denote the recurring of the millions.

I ſhall next propoſe ſome examples for the learner's peruſal and exerciſe.

QUESTIONS.	ANSWERS.
Expreſs in figures, 7 thouſand 5 hundred and 7.	7507
Write 9 hundred million in figures.	900000000.
Write in figures, 9 thouſand 5 million 4 hundred thouſand and 4.	9005400004.
Expreſs in words, 7075654.	7 million and 75 thouſand, 6 hundred and fifty-four.
Write in words, 7497407496784.	
Write in figures, 874 thouſand, 5 million and 27.	

Let

Let 47965000649 6546 be expressed in words.

If more examples are needful, they may easily be proposed by the teacher as he sees occasion.

8. Before I proceed to addition, I shall explain some useful characters of abbreviation, which the learner would do well to make himself master of, as thereby the work will be greatly contracted and frequently the illustrations rendered more elegant and instructive.

$+$ Signifies plus, or added to, as $5+6$ reads 5 plus 6, or 5 added to 6.

$-$ Minus, or subtracted from, as $6-5$ reads 6 minus 5, or 5 subtracted from 6.

$\times$ Into, or multiplied by, as 5×6 reads 5 into 6, or 5 multiplied by 6.

$\div$ Divided by, as $6\div 5$ reads 6 divided by 5; but frequently the divisor is placed below, and the dividend above, with a line between, thus, $\frac{6}{5}$, or thus, $\frac{5\times 6}{3}$, which last reads 5 multiplied by 6 and divided by 3.

$=$ Equal to, as $5+6=11$, is 5 plus 6 is equal to 11.

$:$ and $::$ are proportioned to, as $5:6::10:12$ reads 5 is to 6 as 10 is to 12.

$\sqrt{}$. $\sqrt[3]{}$ and $\sqrt[4]{}$ denote the square, cube, and biquadrate roots, to be taken respectively of such quantities as they are set before; thus $\sqrt{746}$ shews, that the square root of 746 is to be taken, and $\sqrt[3]{74+65}$, that the cube-root of $74+65$ is to be extracted.

ADDITION.

9. WE design to treat of addition, subtraction, multiplication, and division, in whole or integral numbers, being convinced from experience, that the learner, previously acquainted with these, will proceed with more certainty, expedition, and ease, when he comes to their application in quantities of *different denominations*.

De-

DEFINITION.

10. Addition is the finding the ſum total or aggregate, of any number of given ſums.

RULE.

11. *Set down the ſeveral given numbers under each other, and in ſuch order, that the units place make an upright perpendicular column, the place of tens another, &c. then draw a ſtroke below them, and beginning with the units place, reckon it up to the top, and obſerve how many tens are in the whole amount; put down the exceſs, and carry the number of tens contained, to the place of tens, adding it up in like manner as the units. The exceſs being placed down, you carry to the hundreds, &c. till the whole is gone over, and when the laſt column is added, you place it all down (having no other quantity to carry the number of tens to), and the line thus obtained, is termed the* Sum.

EXAMPLE.

Required the ſum of 768476, 476483, 47647, 846892, 46894, and 9678?

This example, placed according to the rule, will ſtand as below.

```
     768476
     476483
      47647
     846892
      46894
       9678
    -------
Sum 2196070
    -------
```

ILLUSTRATION.

12. Beginning at the units place, I ſay 8 and 4 is 12, and 2 is 14, and 7 is 21, and 3 is 24, and 6 is 30; now there are three tens in 30 and 0 over, therefore place 0 down below the units place, carrying forward the 3 tens, or 3 to the place of tens, we have $3+7+9+9+4+8+7=47$; now, in 47 there are 4 tens and an exceſs of 7, which ſet down, carrying forward 4 (or 400) to the place of hundreds, where $4+6+8+8+6+4+4=40$; put 0 down and carry 4 (which is 4000) to

to the place of thousands, then $4+9+6+6+7+6+8=46$; place the 6 down and carry 4 (being 40000) to the place of tens of thousands, which will be $4+4+4+4+7+6=29$; carry forward 2 (or 200000) and place the nine down, therefore $2+8+4+7=21$, which place down, as being the last, and no other column to carry more to; hence the whole amount of these proposed quantities is 2196070; and having illustrated thus plainly the rule, I hope the reason thereof will appear evident to any who understands our received method of notation.

EXAMPLES.

13.	746749	74976	7299846
	29478	16274	456295
	67456	24379	268465
	127647	27564	164864
	46072	21646	264684
	106407	46946	467464
	42084	75647	946567
	8464	7284	746946
	1648	346	484647
Sum	1176005		

What is the sum of $746876+4927+7646+147+674+647+468+746$? Answer 762131.

What is the sum of $8647+4646+2694+9674+7293+5692+1769+219$? Answer 40634.

Required the sum of $47984+84653+749+9465+79624+6749+7490+6508+5649+7946+807$? Answer 257624.

Required the aggregate or sum total of $79462+4907+84976+74949+12000+5746+58+75+8+111+432+98746741$? Answer 99009465.

What is the whole amount or sum of $74675+675+678+4656+652654+579084+4658$ $7549+746565+4968474+654658$? Answer 54269668.

What is the sum total of the following given quantities, *viz.* 4 thousand 7 hundred and 5, six hundred and

7, 9 thousand and 63, twelve million three hundred thousand and 3 hundred, 9 hundred thousand million and 56978? Answer 900012371653.

14. There are several ways of proving addition; but the best way to try if the work be right, in my opinion, is, by adding it twice over, *viz.* beginning at the bottom and proceeding upwards, then from the top downwards, and if these agree, we conclude the work is right.

SUBTRACTION.

DEFINITION.

15. SUbtraction is the deducting one number from another, or finding the difference between any two given sums. The greater number is termed the *minuend*, the less the *subtrahend*, and what remains after subtracting, the *difference*.

RULE.

16. *Place the greater number above, with the less under, keeping units under units, ten under tens,* &c. *(per article* 11.*) then beginning with the units place, take the under digits from those above, placing the differences below their respective columns; but when the figure in the uppermost line is less than the one below it, borrow ten, which add to the figure above, taking the digit in the lower line from the sum, and carrying one to the next figure in the undermost line, so proceeding till the whole is gone over, and then will the difference be obtained.*

EXAMPLE.

From 8746476 termed the *minuend*,
Take 6871584 called the *subtrahend*.

Answer 1874892 denominated the *difference*.

ILLUSTRATION.

If 4 is taken from 6, there is two left, which place under the units; but 8 from 7 cannot be taken, therefore I borrow 1 from the 4 in the hundreds place, (and because of the tenfold ratio, article 3, it becomes ten units to add to the place of tens), saying, 8 from 7 I cannot, but 8 from 17 and 9 remains; which set down in the place of tens, I proceed by adding 1 to the digit in the lower line, and in the place of the hundreds, instead of the 1 I borrowed from the 4 above; or, if none is carried, I consider the 4 above as only 3, because I supposed one to be taken from it, in either case we have the same conclusion, for 1 to 5 is 6, and 6 from 4 I cannot, but 6 from 14 and 8 remains, or 5 from 3 I cannot, but 5 from 13 and 8 remains also: Thus proceed throughout the whole, which will need no further illustration. The proof is by adding the difference to the subtrahend; and the sum, if right, will be the same as the minuend.

EXAMPLES.

From	749678	749076	349708
Take	476948	473479	36497
Difference	272730	275597	313211
Proof	749678	749076	349708

From	174964	75473	496794
Take	84764	42946	147946
Difference			
Proof			

From 7687 take 96. Answer 7591.
From 7694 take 1769. Answer 5925.
From 9 hundred thousand take 9. Answer 899991.
From 5 hundred and 46 million take 92 thousand and 17. Answer 545907983.

From

From 74 thouſand million take 465. Anſwer 73999999535.

Take 3 from 30 thouſand. Anſwer 29997.

Required the difference between 5 thouſand and 6 million and 87497465849? Anſwer 82491465894.

MULTIPLICATION.

DEFINITION.

19. THIS rule diſcovers how much the ſum of a quantity is, taken any given number of times, and is therefore a compendious method of addition; for if the ſum of the digit 9 is taken 8 times, it will be $9+9+9+9+9+9+9+9=72$ but $9\times 8=72$ alſo. Queſtions of this kind may eaſily be anſwered in multiplication, which would be attended with a great deal of trouble if performed by addition, and others would often occur altogether impracticable.

The following table of products is abſolutely neceſſary to be got by heart, in order to facilitate the buſineſs of multiplying.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
	4	6	8	10	12	14	16	18	20	22	24
		9	12	15	18	21	24	27	30	33	36
			16	20	24	28	32	36	40	44	48
				25	30	35	40	45	50	55	60
					36	42	48	54	60	66	72
						49	56	63	70	77	84
							64	72	80	88	96
								81	90	99	108
									100	110	120
										121	132
											144

Obſerve, that the uppermoſt line contains the numbers, 1, 2, 3, &c. to 12, the ſecond is their product by 2, the third their product by 3, &c.

20. To-

20. *To multiply any whole number by a single digit.*

RULE. *Set the multiplier under the units place of the multiplicand, and find the product of every figure in the multiplicand with the multiplier, one after another, carrying one for every ten, setting down the excess as in addition.*

EXAMPLES.

Multiply	749647	called the *multiplicand*,
By	4	named the *multiplier*.
The answer	2998588	termed the *product*.

ILLUSTRATION.

21. I say 4×7 is 28, that is eight to put down and 2 to carry, and $4 \times 4 = 16$, with the 2 carried is 18, set down 8 and carry 1, also $4 \times 6 = 24$, and 1 is 25, put down 5 and carry 2, then $4 \times 9 = 36$, and 2 is 38, setting down the 8 and carrying 3, saying $4 \times 4 = 16$, and 3 is 19, putting down 9 I carry 1, and $4 \times 7 = 28$, and 1 is 29, which I place down in full, see art. 12. hence the whole product becomes 2998588.

Multiply	746875	Multiply	76748671
By	5	By	8
Product	3734375	Product	613989368

Multiply	476946	Multiply	474948
By	4	By	7

Multiply 7490765 by 9. Answer

Multiply 56748 by 7. Answer

Required the product of 749678 by 6? Answer

22. To

22. *To multiply any whole number, by* 10, 100, 1000, &c.

RULE. *Set down the multiplicand with the ſame number of cyphers annexed thereto, as there are contained in the multiplier* *.

EXAMPLES.

Multiply 74765 by 10. Anſwer 747650.
Multiply 9476467 by 100. Anſwer 947646700.
Multiply 594540 by 1000. Anſwer 59454000.

23. *To multiply any whole number by a ſingle digit with cyphers annexed thereto.*

RULE. *Put down the given cyphers on the right-hand ſide of the product of the digit* †.

EXAMPLES.

Multiply	7656	Multiply	768465
By	400	By	30000
Product	3062400	Product	23053950000

Multiply 470746 by 5000. Anſwer 2353730000.
Multiply 970456 by 80. Anſwer 77636480.
Multiply 746593 by 900. Anſwer 671933700.

24. *To multiply any two whole numbers into each other.*

RULE. *Find the product of every digit in the multiplier, (by art. 20.) minding to place the right-hand figure of every line under its reſpective multiplying digit; the ſum of theſe ſeveral lines ſo placed is the product.*

Ex-

* The reaſon of this rule will appear evidently, if what is advanced at article 3, be clearly underſtood.

† This rule is founded upon the laſt, together with that laid down, article 20.

EXAMPLE.

Multiply	47475		
By	53004		
	189900	the product of	4 *per* art. 20.
	142425000	the product of	3000 *per* art. 23.
	2373750000	the product of	50000 *per* art. 23.
	2516364900	the product of	53004

If the cyphers are omitted in the above process, the work will stand as directed *per* rule.

47475	
53004	
189900 *	
142425	
237375	
2516364900	

0
0 X 0
0

25. The common method of proving multiplication, is by casting out the nines, and is effected in the following manner, viz. beginning with the multiplicand, we say, 4 added to 7 is 11, which is two more than 9, and 2 to 4 is 6, and 7 make 13, which is an excess of 4 above 9, then 4 and 5 make 9, that is nothing above; therefore we place 0 down in the cross adjacent to the example, and proceed to the multiplier, where $5+3+4=12$, and 12 is 3 over 9, therefore the excess 3 is placed opposite the 0 in the said cross; then must these two figures be multiplied together, and the 9s cast out, placing the excess in the cross at the top, which (in the present case) will be $3\times 0=0$, and the 9s in 0 is no times

* The cyphers in this line are to be retained, otherwise the product would be only one hundredth part of its just value, per art. 3.

times and 0 over, this 0 ſet down as directed, and caſt the 9s out of the product thus, $2+5+1+6=14$, which is 5 over, and $5+3+6=14$, that is an exceſs of 5, then $5+4=9$, which is nothing over, being the ſame with the other 0 placed above, and is therefore put below it to denote their agreement, from which we conclude the work is right, though not infallibly ſo; for the ſame figures ſet in any order, will afford the ſame concluſion by caſting out the 9s. The true way of proving multiplication is by diviſion only; but if every line be proved as you proceed, and, laſtly, the whole product, we may very ſafely conclude the work is right *.

26. It now remains, that we propoſe a few examples for the inſpection and exerciſe of the learner.

Multiply

* A demonſtration of the above method.

It is the peculiar property of the digit 9 to divide any of the other digits when cyphers are annexed, and leave a remainder equal to that very digit; for 80 divided by 9 and 8 remains, 700 divided by 9 and 7 remains, &c. Moreover, if the 9s are caſt out of any number, and the ſame number divided by 9, the remainders will be equal; for 76 divided by 9 and 4 remains, alſo $7+6=13$, and the exceſs is 4 likewiſe. But again, if any number be taken in parts, and multiplied by ſome other number, the products of theſe parts with that other number, will equal the product of the whole entire number, when multiplied by the ſaid number. For let 54 and 2 be parts of 56, and let theſe parts be multiplied with 6, then will $54\times6=324$, and $2\times6=12$; but $324+12=336$, the ſum of theſe parts when multiplied by 6; now $56\times6=336$ alſo. It muſt be obſerved, that whatever number is multiplied by 54, the product will be diviſible by 9, becauſe 54 is a multiple of 9, (that is, if 9 is multiplied into ſome other integral quantity, it will be 54) what exceſs therefore ariſes from dividing by 9, muſt be from the product of the other part, viz. $2\times6=12$, which has 3 over; but if the whole product is divided by 9, (that is $336\div9$), there remains 3 alſo. *Q. E. D.*

	Multiply 464765	Multiply 87456
	By 13074	By 4326
3	1859060	524736
6 X 5	3253355	174912
3	1394295	262368
	464765	349824
	6076337610	378334656

What is the product of 751900368 by 51. Answer 38346918768.

Required the product of 402097316 by 195. Answer 78408976620.

Required the product of 46213 into 9832. Answer 454366216.

What is the product of 5697487 by 98328. Answer 560222501736.

DIVISION.

DEFINITION.

27. THIS rule is the reverse of Multiplication, and therefore is a compendious method of performing subtraction. By it we discover how many times one number is contained in another, and consists of three terms, the *divisor*, or number divided by, the *dividend*, or number divided, the *quotient*, or number of times the divisor is contained in the dividend.

RULE.

Ask how many times the divisor is contained in a like number of figures on the left hand side of the dividend; if a like number be too little, take one more, and place the digit expressing the number of times in the quotient, with which multiply the divisor, setting the product under the said figures in the dividend, and taking the difference,

to which bring down the next figure in the dividend, and aſk how often the diviſor is contained therein, place the figure denoting the number of times in the quotient, by which multiply the diviſor, &c. ſo proceeding till all the figures in the dividend are exhauſted, and then is the work ended.

EXAMPLE.

How many times is 79 contained in 7694763?

```
Diviſor. Dividend. Quotient.
   79) 7694763 (90000
       7110000
       -------
   79)  584763 ( 7000
        553000
        ------
   79)   31763 (  400
         31600
         -----
   79)     163 (    2
           158   -----
           ---   97402
Remainder    5   -----
           ---
```

ILLUSTRATION.

Having placed the diviſor and dividend as above, I aſk how many times 79 is contained in 7694763? and find it 90000, which multiplied by 79, and the product placed below the dividend, the difference is had by ſubtraction, equal 584763, and the 79s in it is 7000, which multiplied by 79, and the product taken from 584763, there remains 31763, in which 79 is contained 400 times, being multiplied by 79, and the product taken from the laſt difference, leaves 163, and the 79s contained therein, is 2 times, which multiplied by 79, and the product taken from 163, gives the laſt remainder 5.

I ſhall now ſhew the method of performing the ſame example according to the rule.

Diviſor.

Divisor. Dividend. Quotient.

```
79) 7694763 (97402
    711····
    ———
     584
     553
     ———
      317
      316
      ———
       163
       158
       ———
         5
       ———
```

ILLUSTRATION.

Becauſe 79 cannot be had in the firſt two figures, I take another to them, ſaying, How many times can it be had in 769? which, upon trial, I find to be nine times; and 9 multiplied into 79 is 711, this placed below 769, and ſubtracted from it, the difference is 58, to which bring down the next figure in the dividend, viz. 4, and it then becomes 584, and the 79s therein is 7 times; and 7×79 is 553, now the difference between 584 and 553 is 31, to which bring down 7, ſaying, the 79s in 317 is 4 times, and the product of 4 by 79, taken from 317, gives a difference of 1 only, and 6 brought to it makes it 16; but the 79s in 16 is 0 times, therefore I place the 0 in the quotient, and bring down another figure from the dividend, viz. 3, and then aſk, how many times 79 in 163? which I find to be 2 times, the product of which with 79, taken from 163, leaves a remainder of 5; and the number of times the 79s are contained in the ſeveral quantities, put in the quotient as you proceed, will be 97402, which agrees with the former method of performing the work, where, we may obſerve, there is no other difference in theſe operations, than an omiſſion of the cyphers in the latter caſe.

EXAMPLES.

```
17) 456765 (26868 Anſwer.
  + 34....         17
    ----       ------
    116        188076
  + 102         26868
    ---             9 remainder.
     147       ------
   + 136       456765 proof by multiplication.
     ---       ------
      116
    + 102
      ---
       145
     + 136
       ---
       + 9
    ------
    456765 proof by addition.
    ------

13074) 6076337610 (464765
       52296
       -----
        84673
        78444
        -----
         62297
         52296
         -----
         100016
          91518
         ------
          84981
          78444
          -----
           65370
           65370
           -----
```

```
87456) 378334656 (4326
       349824
       ------
        285106
        262368
        ------
         227385
         174912
         ------
          524736
          524736
          ------
```

Divide 38346918768 by 51. Anſwer 751900368.

Required the quotient, when the diviſor is 9832, and the dividend 454366216? Anſwer 46213.

Let 549739125656 be divided by 5697487. Anſwer 96488.

Required how many times 402097316 can be had in 78408976620? Anſwer 195.

28. The proof of diviſion is either by addition or multiplication. If the quotient and diviſor are multiplied together, and the remainder (if any) taken in, the product will be the dividend. And if the products of the diviſor, with the ſeveral quotient digits, (in a preceding example which is proved both ways, they are marked +) are added together, the ſum will be equal to the dividend alſo.

29. There is a very uſeful and compendious way of diviſion, commonly called *ſhort diviſion*, by which any diviſor not exceeding 12, may be managed.

EXAMPLE.

```
Diviſor. Dividend.
      4) 6876523
         -------
Quotient 1719130 3 remainder.
         -------
```

ILLU-

ILLUSTRATION.

The diviſor and dividend being placed as above, I ſay, the 4s in 6 is once and 2 over, the 4s in 28 is 7 times and 0 over, the 4s in 7 is once and 3 over, the 4s in 36 is nine times and 0 over, the 4s in 5 is once and 1 over, the 4s in 12 is 3 times and 0 over, and the 4s in 3 is 0 times and 3 remains; hence the quotient is 1719130.

In the method before us, we keep the remainder in mind, ſuppoſing it to ſtand before the next figure, and in the other kind of diviſion the difference is ſet down, and the next figure in the dividend brought to it. Hence, in the methods of operation, the difference conſiſts in keeping in mind in the one way, what, in the other compoſes that part of the work called the chain.

Diviſor. Dividend.		
	5) 764678	12) 1164764
Quotient.	152935 3	97063 8

Divide 74965980 by 12. Anſwer 6247165.
Divide 8476596 by 9. Anſwer 941844.

30. Any multiple of numbers, not exceeding 12, may be performed by this method.

EXAMPLES.

Divide 746546595 by 18.

Becauſe $6 \times 3 = 18$, therefore if we firſt divide by one of the components and then the other, the laſt quotient will give the anſwer. Thus,

6) 746546595

3) 124424432 3

41474810 2 *

Divide

* It will often be neceſſary to find what the remainder would be, had the work been performed at one operation; to do which, let the laſt remainder be multiplied into the firſt diviſor, taking in the firſt remainder thus, $2 \times 6 = 12$, and 3 is 15.

		Quotient.	Remainder.
Divide 769476 by 63.	Anſ.	12213	57.
Divide 769764 by 144.	Anſ.	5345	84.
Divide 7676844 by 48.	Anſ.	159951 7	28.

31. In any diviſion where there are cyphers to the right hand, they may be left out or ſtruck off, daſhing as many places of figures off on the right-hand ſide of the dividend.

EXAMPLES.

Divide 467652 by 20.
2|0) 46765|2
Quotient 23382 12 remainder.

Divide 74964 by 500.
5|00) 749|64
149 464

Divide 4765940 by 240.
Divide 54897400 by 4800.
How many times will 7200 be contained in 756740000?

The reaſon of theſe contractions will be evident, if compared with article 23d and 24th, being the reverſe of theſe rules.——Many more contractions might be added, but theſe are the moſt uſeful and practical; we ſhall therefore now paſs them by, and perhaps advance ſomething more on this ſubject, when the pupil will be better qualified for an undertaking of the kind.

ADDITION of COMPOUND QUANTITIES.

GENERAL RULE.

32. *Collect all the quantities of one ſpecies together, dividing the ſum by ſo many of this denomination as make one of the next ſuperior, placing down the exceſs, and carrying the quantity expreſſed in the quotient, to the next ſuperior denomination.*

Of

Of MONEY.

Note, that	4 farthings make	1	penny.
	12 pence	1	ſhilling.
	20 ſhillings	1	pound.
	21 ſhillings	1	guinea.

Alſo pounds are diſtinguiſhed by £. ſhillings by *S.* and pence by *D.* or *d.* the farthings are denoted as followeth, $\frac{1}{4}$ is a farthing, $\frac{2}{4}$ or $\frac{1}{2}$ is a halfpenny, and $\frac{3}{4}$ three farthings.

EXAMPLES.

£.	s.	d.	£.	s.	d.	£.	s.	d.
746	14	$7\frac{1}{4}$	749	10	0	74	17	$11\frac{1}{2}$
76	16	$10\frac{1}{2}$	14	17	$6\frac{1}{2}$	17	14	$10\frac{1}{4}$
19	16	$4\frac{1}{2}$	84	16	7	84	16	$9\frac{1}{4}$
14	17	$10\frac{1}{4}$	84	17	$10\frac{1}{2}$	84	12	$10\frac{3}{4}$
216	17	$10\frac{1}{4}$	97	18	$9\frac{3}{4}$	87	9	$9\frac{1}{2}$
19	16	$4\frac{1}{2}$	84	19	$10\frac{1}{4}$	21	8	$8\frac{1}{4}$
26	14	$11\frac{1}{2}$	92	17	$11\frac{1}{4}$	34	14	$10\frac{3}{4}$
	16	9	12	14	11	94	16	$11\frac{1}{4}$
1122	11	$7\frac{3}{4}$						

ILLUSTRATION.

Beginning with the farthings, ($\frac{1}{2}$, where-ever it occurs, is accounted 2 farthings), I ſay 2 and 2 is 4, and 1 is 5, and 1 is 6, and 2 is 8, and 2 is 10, and 1 is 11, and (4 farthings being 1 penny) the 4s in 11 is 2 times and 3 over; therefore $\frac{3}{4}$ is ſet down, and 2 pence carried to the column of pence; then in the units place ſay, $2+9+1+4+4+7=27$, and every 1 in the place of tens is accounted 10. that is, 27 and 10 is 37, and 10 is 47, and 10 is 57, and 10 is 67 pence, and (becauſe 12 pence make 1 ſhilling) the 12s in 67 is 5 times and 7 over, that is 5 *s.* 7 *d.* the 7 we ſet down under the pence, and the 5 is carried to the units place

place of the ſhillings, ſaying, $5+6+4+6+7+7+6+6+4=51$; and beginning at the top, I ſum the tens, ſaying, 51 and 10 is 61, and 10 is 71, and 10 is 81, and 10 is 91, and 10 is 101, and 10 is 111, and 10 is 121, and 10 is 131 ſhillings, which divided by 20 (the ſhillings in a pound) gives 6 *l.* 11 *s.* that is, 11 to put down under the ſhillings, and 6 to carry to the pounds, which are added as whole numbers by art. 12. and therefore need not in this place be further inſiſted upon. Some chuſe to have the learner get a table of pence by heart; but I prefer the method of dividing the ſum of the pence by 12, becauſe it is general, extending to any number how large ſoever.

I ſhall inſert a few examples for the learner's inſtruction; and after ſuch a manner as to require being placed down over again before they can be added, becauſe I have found from experience, that boys are ſometimes at a greater loſs to prepare their queſtions properly, than to add them.

Required the ſum total of 76 *l.* 10 *s.* 146 *l.* 17 *s.* 4 *d.* 5 *l.* 13 *s*, 4 *d.* 12 *s.* $7\frac{1}{2}$ *d.* 16 *l.* 19 *s.* $11\frac{1}{2}$ *d.* 165 *l.* 16 *s.* and 765 *l.* Anſwer 1177 *l.* 9 *s.* 3 *d.*

What is the ſum of 56 *l.* 16 *s.* 9 *d.* 1 *l.* 19 *s.* $4\frac{1}{2}$ *d.* 567 *l.* 17 *s.* $9\frac{3}{4}$ *d.* 36 *l.* 16 *s.* 4 *d.* 76 *l.* 16 *s.* $9\frac{1}{2}$ *d.* 769 *l.* 14 *s.* 676 *l.* 16 *s.* $7\frac{1}{2}$ *d.* 6 *l.* 17 *s.* $9\frac{1}{4}$ *d.* 6926 *l.* 16 *s.* $7\frac{1}{2}$ *d.* 15 *s.* $9\frac{1}{4}$ *d.* and 17 *l.* 19 *s.* $9\frac{1}{2}$ *d.* Anſwer 9139 *l.* 7 *s.* $7\frac{1}{4}$ *d.*

What is the amount of 749 *l.* 16 *s.* $11\frac{1}{4}$ *d.* 467 *l.* 16 *s.* 10 *d.* 19 *l.* 16 *s.* 11 *d.* 74 *l.* 19 *s.* $9\frac{1}{2}$ *d.* 462 *l.* 17 *s.* $4\frac{3}{4}$ *d.* 479 *l.* 11 *s.* $4\frac{1}{2}$ *d.* 467 *l.* 18 *s.* 4 *d.* 469 *l.* 12 *s.* $9\frac{1}{4}$ *d.* 749 *l.* 12 *s.* 49 *l.* $10\frac{3}{4}$ *d.* 45 *l.* 19 *s.* 6 *d.* and 17 *s.* $10\frac{1}{4}$ *d.* Anſwer 4038 *l.* 0 *s.* $8\frac{1}{4}$ *d.*

Bought the following articles, which coſt as below, butter, 1 *s.* $2\frac{1}{2}$ *d.* cheeſe, 5 *s.* 9 *d.* beef, 11 *s.* 9 *d.* mutton, 3 *s.* 6 *d.* veal, 4 *s.* 7 *d.* ducks, 2 *s.* 4 *d.* hens, 1 *s.* $4\frac{1}{2}$ *d.* geeſe, 4 *s.* 6 *d.* ſalmon, 2 *s.* 4 *d.* ſugar, 4 *s.* 6 *d.* tea, 16 *s.* 4 *d.* coffee, 12 *s.* 6 *d.* and chocolate, 1 *l.* 7 *s.* 6 *d.* What is the expence of the whole? Anſwer 4 *l.* 18 *s.* 2 *d.*

The

The ſide of a Ledger conſiſts of the following ſums, required the ſum total? 12 *l.* 19 *s.* 5 *l.* 19 *s.* 4 *d.* 4 *l.* 10 *s.* 6 *d.* 5 *l.* 10 *s.* 9 *d.* 46 *l.* 18 *s.* 4 *d.* 46 *l.* 19 *s.* 84 *l.* 19 *s.* 4 *d.* 94 *l.* 6 *s.* 5 *l.* 19 *s.* 4 *d.* 4 *l.* 16 *s.* 11 *d.* 46 *l.* 12 *s.* 9 *d.* 49 *l.* 12 *s.* 4 *d.* 32 *l.* 14 *s.* 9 *d.* 46 *l.* 19 *s.* 11 *d.* 49 *l.* 12 *s.* 46 *l.* 19 *s.* 4 *d.* 467 *l.* 19 *s.* 4 *d.* 47 *l.* 12 *s.* 84 *l.* 12 *s.* and 45 *l.* 19 *s.* 6 *d.* Anſwer 1231 *l.* 12 *s.* 5 *d.*

A merchant balancing his books, finds he has as follows, viz. In caſh, 1446 *l.* 14 *s.* 4 *d.* broad cloths, 762 *l.* 12 *s.* ſcarlets, 64 *l.* 14 *s.* ſerges, 49 *l.* 13 *s.* 4½ *d.* ſhip, Dolphin, 926 *l.* 12 *s.* ſugars, 144 *l.* 17 *s.* 6 *d.* indigoes, 400 *l.* coffee, 324 *l.* 16 *s.* 2¼, tea, 46 *l.* 19 *s.* 11 *d.* wines, 747 *l.* 16 *s.* by W. Watſon, 176 *l.* 14 *s.* 7 *d.* Robert Loſs, 97 *l.* 17 *s.* 7 *d.* Sam. Simpſon, 707 *l.* 14 *s.* and by John Thompſon, 112 *l.* 19 *s.* 8 *d.* What does the whole amount to? Anſwer 6010 *l.* 1 *s.* 1¾ *d.*

Laid out, half-a-crown, ſeventeen pence, five groats and two pence, thirteen pence, a crown, half-a-guinea, three half crowns, eight groats, twenty-five ſhillings, thirteen pence, five farthings, a quarter-guinea, and eighteen pence; required the ſum in pounds, ſhillings, &c. Anſwer 3 *l.* 5 *s.* 5¼ *d.*

A gentleman dying, left by will, to his eldeſt ſon, three thouſand and 50 pounds; to his younger ſon, a thouſand 5 hundred; to his daughter 7 hundred and 50; to 5 poor relations, each 35 *l.* and to his executors the remainder, which was 265 *l.* I demand the old gentleman's fortune? Anſwer 5740.

An old man's age was required, who ſaid I have 5 ſons and three daughters, between the birth of each of my ſons was two years, between my laſt ſon and firſt daughter 4 years; likewiſe 4 years between each of the reſt; in my 20th year was my firſt ſon born; and that is now the age of my youngeſt daughter, from hence the father's age is required. Anſwer 60 years.

We ſhall, in this place, inſert the tables of the different denominations of the moſt ordinary weights and meaſures, as, without an acquaintance with them, the addition of thoſe weights, &c. cannot eaſily be attained.

In TROY WEIGHT.

24	grains (marked *Gr.*) make	1	pennyweight,
20	pennyweights (*dwt.*)	1	ounce,
12	ounces (*oz.*)	1	pound (marked *lb.*)

By which are weighed gold, ſilver, electuaries, *&c.*

In AVOIRDUPOIS WEIGHT.

16	drams (marked *dr.*) make	1	ounce,
16	ounces (*oz.*)	1	pound,
28	pound (*lb.*)	1	quarter,
4	quarters (*qr.*)	1	hundred weight,
20	hundred weight (*cwt.*)	1	ton.

Groceries, iron, lead, *&c.* are weighed by this weight.

In APOTHECARIES WEIGHT *.

20	grains (marked *gr.*) make	1	ſcruple,
3	ſcruples (℈)	1	dram,
8	drams (ʒ)	1	ounce,
12	ounces (℥)	1	pound, (*lb.*)

In LONG MEASURE.

3	barley-corns make	1	inch,
12	inches	1	foot,
3	feet	1	yard,
$5\frac{1}{2}$	yards	1	pole or perch,
40	poles	1	furlong,
8	furlongs	1	mile,
3	miles	1	league,
20	leagues	1	degree.

In CORN MEASURE.

2	gallons make	1	peck,
4	pecks	1	buſhel,
8	buſhels	1	quarter,
5	quarters	1	load.

* Apothecaries buy drugs by Avoirdupois weight, but ſell by this. We may likewiſe obſerve, that 17 *lb.* Troy, is nearly 14 *lb.* Avoidupois.

In WINE MEASURE.

2	pints make	1 quart,
4	quarts	1 gallon,
63	gallons	1 hogſhead,
4	hogſheads (or 2 butts)	1 tun.

In ALE *and* BEER MEASURE.

2	pints make	1 quart,
4	quarts	1 gallon,
8	gallons	1 firkin of ale,
9	gallons	1 firkin of beer,
4	firkins	1 barrel,
2	barrels	1 hogſhead.

In TIME.

60	ſeconds make	1 minute,
60	minutes	1 hour,
24	hours	1 day,
7	days	1 week,
4	weeks	1 month,

12 callender months, or 13 lunar months, or 365 days, 1 year.

In CIRCULAR MOTION.

60	thirds (marked ‴) make	1 ſecant,
60	ſeconds (″)	1 minute,
60	minutes (′)	1 degree,
30	degrees (°)	1 ſign,

12 ſigns, or 360 degrees the whole circumference of a circle.

EXAMPLES.

lb.	*oz.*	*dwt.*	*gr.*	*tons*	*cwt.*	*qr.*	*lb.*
864	10	17	22	479	14	2	18
769	7	19	14	179	12	1	19
876	11	16	22	147	16	3	18
146	10	18	23	129	14	1	18
87	6	19	18	72	17	2	24
176	10	14	24	114	16	3	25
84	11	12	19	72	19	2	19
24	10	18	16	121	17	3	25

cwts.	*qr.*	*lb.*
749	1	14
472	3	27
84	1	18
84	1	24
87	2	19
716	3	21
76	1	24
16	3	19
24	1	27

lb.	*oz.*	*dr.*
124	10	00
147	14	6
126	15	14
216	14	12
846	13	14
741	14	15
24	10	11
17	9	7
24	7	12

tons	*cwt.*	*qr.*
7	17	2
9	18	1
12	17	0
7	19	1
14	16	3
7	9	2
8	14	1
4	12	0
7	14	0

miles	*f.*	*p.*	*yds*
749	7	14	4
714	6	24	$4\frac{1}{2}$
747	7	29	3
146	5	18	$4\frac{1}{2}$
96	4	39	5
164	6	24	5
84	1	25	$1\frac{1}{2}$
79	5	24	3

°	′	″
33	17	21
49	59	47
17	14	56
36	17	14
39	14	25
47	16	28
14	17	25
27	17	54

tons	*hhds.*	*gll.*	*qts.*
742	3	52	3
76	1	62	2
76	2	61	2
19	2	53	3
126	1	42	1
27	0	16	2
24	1	24	3
7	3	19	2
1101	2	18	2

34. It would be needleſs to multiply examples under all the different kinds of weights, meaſures, &c. as theſe can eaſily be enlarged upon by the teacher, or judicious learner; only I would obſerve to the latter, that, in propoſing examples to himſelf, he is always to conſider the higheſt denomination (or that towards his left hand) as whole numbers, and, in any of the others, he muſt not ſet down a quantity ſo large as to make one or more of the next ſuperior denomination: for inſtance in the laſt example, at gallons no number ought to exceed 62, becauſe 63 gallons make one hogſhead. To render the method of adding the different ſpecies of weights, meaſures, &c. as eaſy as poſſible, I ſhall give

an

an illustration of the last example agreeable to the general rule, art. 32. And beginning with the quarts, we have 2 + 3 + 2 + 1 + 3 + 2 + 2 + 3 = 18 quarts, which divided by 4, gives 4 gallons and 2 quarts, the quarts being set down, we carry 4 to the gallons, where 4 + 9 + 4 + 6 + 2 + 3 + 1 + 2 + 2 = 33, in a convenient place of the paper or slate, place down 3, and carry the other 3 to the place of tens at the said gallons, and 3 + 1 + 2 + 1 + 4 + 5 + 6 + 6 + 5 = 33, which placed before the other 3, becomes 333, now the 63s in 333, is 5 times and 18 over; that is 5 hogsheads 18 gallons; therefore set down 18 and carry 5, and 5 + 3 + 1 + 0 + 1 + 2 + 2 + 1 + 3 = 18, and the 4s in 18 is 4 times and 2 over, viz. 4 tuns and 2 hogsheads, the 2 place under hogsheads, and carry the 4 to tuns, which are added as directed art. 12.

SUBTRACTION of COMPOUND QUANTITIES.

35. RULE. *In any denomination, when the quantity in the lower line is more than that above, borrow so many as make one of the next superior kind, which add to the uppermost line, and the difference between this sum and the quantity below place down, observing to carry an unit to the next superior denomination as often as you have occasion to borrow.*

Of MONEY.

EXAMPLES.

From	764	16	$9\frac{1}{2}$	794	19	$7\frac{1}{2}$
Take	72	17	$7\frac{3}{4}$	72	16	$10\frac{1}{4}$
Difference	691	19	$1\frac{3}{4}$	722	2	$9\frac{1}{4}$
Proof	764	16	$9\frac{1}{2}$			

ILLU-

ILLUSTRATION.

Now, in example first, $\frac{3}{4}$ from $\frac{1}{2}$ I cannot, but borrowing 4, it is $4+2=6$, that is 3 from 6 and $\frac{3}{4}$ remains, which put down, I carry the 4 farthings, being 1 penny, to the pence, saying $1+7$ is 8, and 8 from 9 and 1 remains, which put down, I have none to carry; but 17 from 16 I cannot, I therefore borrow 20, and $20+16=36$, then 17 from 36 and 19 remains, which place under shillings, carrying 1 to the pounds, they are managed by art. 16. and the proof is had by adding the subtrahend to the difference.

EXAMPLES.

	£.	s.	d.	£.	s.	d.	£.	s.	d.
From	749	17	$4\frac{1}{2}$	746	14	0	84	14	9
Take	112	17	$9\frac{3}{4}$	84	15	$9\frac{1}{4}$	17	19	$10\frac{1}{4}$
Diff.									
Proof									

	£.	s.	d.	£.	s.	d.	£.	s.	d.
Borrowed	747	17	$9\frac{1}{4}$	746	17	9	874	19	$0\frac{1}{4}$
Paid	74	18	$4\frac{3}{4}$	614	19	$11\frac{1}{4}$	176	18	$7\frac{1}{4}$
To pay									

	£.	s.	d.	£.	s.	d.	£.	s.	d.
Lent	749	14	$9\frac{1}{2}$	749	14	11	746	16	$9\frac{1}{4}$
Received	646	19	$10\frac{1}{4}$	478	19	$10\frac{1}{2}$	72	19	$10\frac{1}{2}$
To receive									
Proof									

	£.	s.	d.	£.	s.	d.	£.	s.	d.
Received	74	19	$4\frac{1}{2}$	746	16	$7\frac{3}{4}$	876	17	$9\frac{1}{4}$
Paid	16	16	$9\frac{3}{4}$	14	19	$7\frac{1}{2}$	146	19	$11\frac{3}{4}$
Remains									
Proof									

From 1 *l.* take $1\frac{1}{4}$ *d.* Answer 19 *s.* $10\frac{3}{4}$ *d.*

From 54 thousand and 5 pounds take 9 *l.* 12 *s.* $6\frac{1}{2}$ *d.* Answer 53995 *l.* 7 *s.* $5\frac{1}{2}$ *d.*

Borrowed 50 pounds; paid 12 *l.* 14 *s.* 6 *d.* What's to pay? Answer 37 *l.* 5 *s.* 6 *d.*

From 92 thousand pound take a farthing. Answer 91999 *l.* 19 *s.* $11\frac{3}{4}$ *d.*

	£.	s.	d.
Borrowed	750	0	0
Paid at different times	124	12	0
	12	17	$9\frac{1}{4}$
	14	16	$11\frac{1}{2}$
	24	17	$10\frac{3}{4}$
	527	16	$4\frac{3}{4}$
Paid in all			
Yet to pay	44	18	$11\frac{1}{4}$

	£.	s.	d.
Borrowed	749	12	6
Paid at different times	49	19	$7\frac{1}{2}$
	124	16	7
	416	17	8
	56	17	9
	84	19	$7\frac{3}{4}$
Paid in all			
Yet to pay			

Borrowed 1749 *l.* 14 *s.* 6 *d.* + 52 *l.* 17 *s.* $4\frac{1}{2}$ *d.* + 78 *l.* 12 *s.* + 192 *l.* 17 *s.* 10 *d.* + 76 *l.* 10 *s.* 6 *d.* + 72 *l.* 19 *s.* 11 *d.* + 78 *l.* + 100 *l.* + 192 *l.* 12 *s.* 9 *d.* Paid 749 *l.* + 10 *l.* 12 *s.* + 73 *l.* 19 *s.* 6 *s.* + 144 *l.* 19 *s.* 4 *d.* + 49 *l.* 19 *s.* 6 *d.* + 78 *l.* 19 *s.* 6 *d* + 74 *l.* 16 *s* + 19 *l.* 12 *s.* 9 *d.* + 92 *l.* 17 *s.* 6 *d.* + 96 *l.* + 17 *l.* 19 *s.* 10 *d.* What's remaining to pay? Answer 1185 *l.* 8 *s.* $11\frac{1}{2}$ *d.*

Of

Of WEIGHTS and MEASURES.

EXAMPLES.

	lb.	*oz.*	*dwt.*	*gr.*
Bought	754	10	14	20
Sold	456	4	16	22
To sell	298	5	17	22
Proof	754	10	14	20

lb.	*oz.*	*dwt.*	*gr.*
72	10	17	17
19	10	16	21

oz.	*dwt.*	*gr.*
72	17	14
66	19	21

tons	*cwt.*	*qr.*	*lb.*
745	17	3	19
75	17	3	24
669	19	3	23

cwt.	*qr.*	*lb.*
17	1	14
16	2	21

lb.	*oz.*	*dr.*
72	7	6
16	14	12

pipes	*gll.*	*qr.*
7	44	3
5	56*	2
1	51	1
7	44	2

gll.	*qr.*	*pt.*
3	3	1
2	2	1

miles	*f.*	*p.*	*yds.*
72	4	14	$4\frac{1}{2}$
16	5	21	5

Take 17 grains of silver from 6 ounces. Answer 5 *oz.* 19 *dwt.* 7 *gr.*

Take 14 *lb.* Avoirdupois from 14 hundred weight. Answer 13 *cwt.* 3 *qr.* 14 *lb.*

Take

* In this and the former examples, it is better to take the quantity below from the number we borrow, and then add what remains to the upper line, which will give the difference: thus 56 from 44 I cannot, but 56 from 63 and 7 remains, and 7 and 44 make 51, the true difference.

Take 4 gallons of wine from 4 butts. Answer 3 *b.* 1 *hhd.* 59 *gll.*

From 3 miles take 4 yards. Answer 2 *m.* 7 *f.* 39 *p.* $1\frac{1}{2}$ *yd.*

From 1 year, or 365 days, take 40 minutes.

	m.	*w.*	*d.*	*h.*	*m.*
Answer	12	3	6	23	20.
		days		*h.*	*m.*
		364		23	20

Suppose a child born in the year 1739, married in the year 1762, how old was he when married? how long since his marriage? and how old is he this present year 1770?

Answer { 23 years of age when married.
8 years since his marriage.
31 years his present age.

Suppose the foundation of a castle laid in the year of Christ 400, finished in 451, destroyed in 946, the foundation relaid in 1264, refinished in 1300, and is now standing, viz. in 1770, I demand how long in building? how long it stood? how long in ruins? how long in rebuilding? how long since rebuilt? and how long since the first foundation was laid? Answer. In building, 51 years; it stood, 495 years; in ruins, 318 years; in rebuilding, 36 years; since rebuilt, 470 years; since the foundation was first laid, 1370 years.

MULTIPLICATION of COMPOUND QUANTITIES.

RULE.

Multiply each denomination in the multiplicand by the multiplier, finding how many of the next superior kind the product makes, placing down the excess, and carrying the quantity contained to the next greater denomination, so proceeding throughout the whole.

Ex-

EXAMPLES.

	£.	s.	d.	
Multiply	47	17	$4\frac{1}{2}$	by 5.
			5	
£.	239	6	$10\frac{1}{2}$	product.

ILLUSTRATION.

Beginning with the lowest denomination, I say 5 times 1 is 5, and because 5 halfpennies is $2\frac{1}{2}$*d.* I set down $\frac{1}{2}$ and carry 2, saying $5 \times 4 = 20$, and 2 carried is 22; then because 22 pence make 1 *s.* 10 *d.* I place down 10 *d.* and carry 1, then say 5 times 7 is 35, and 1 more is 36, in a convenient place of the paper or slate. I set down 6 and carry 3 to the tens place of the shillings, and say 5×1 is 5, and 3 is 8, which placed before the 6, it becomes 86 shillings, which is 4 *l.* 6 *s.* I then place the 6 *s.* down, carrying 4 to the units place in the pounds, which is performed agreeable to what is taught article 20. The inspection of the following examples will assist the student in his further enquiries into this useful and practical rule.

	£.	s.	d.
Multiply	74	14	9
By			6
Product	448	8	6

		s.	d.
Multiply		17	$9\frac{1}{2}$
By			7
£.	6	4	$6\frac{1}{2}$

		oz.	dwt.	gr.
Multiply		3	10	21
By				4
lb.	1	2	3	12

	lb.	oz.	dwt.
Multiply	3	8	3
By			5
lb.	18	4	15

cwt.

	cwt.	qr.	lb.		lb.	oz.	dr.
Multiply	13	1	21	Multiply	8	8	8
By			6	By			8
cwt.	80	2	14	lb.	68	4	0

37. If the multiplier exceed 12, its components muſt be found, that is, ſuch numbers as, when multiplied together, ſhall produce the quantity wanted.

£. s. d.
Multiply 7 12 6 by 72.

	£.	s.	d.
Becauſe $8 \times 9 = 72$, therefore	7	12	6
			8
	61	00	0
	9		
	£. 549	anſwer.	

cwt. qr. lb.
Multiply 2 1 14 by 75.

	cwt.	qr.	lb.
Becauſe $5 \times 5 \times 3 = 75$, therefore	2	1	14
			5
	11	3	14
			5
	59	1	14
			3
cwt.	178	0	14 anſwer.

Multiply 74 *l.* 16 *s.* 4 *d.* by 567.

$9 \times 9 \times 7 \times 74$ *l.* 16 *s.* 4 *d* = 42421 *l.* 1 *s.* 0 *d.* anſ.

Multiply 17 *s.* $9\frac{1}{2}$*d.* by 576.

$8 \times 8 \times 9 \times 17$ *s.* $9\frac{1}{2}$*d.* = 512 *l.* 8 *s.* 0 *d.* anſ.

38. But

38. But if the multiplier be ſuch as cannot be expreſſed by the continued multiplication of components, take ſome numbers, the product of which may come near the value of the quantity given, then with the defect or exceſs, multiply the multiplicand, the ſum or difference of theſe products reſpectively, will give the true anſwer.

EXAMPLES.

	£.	*s.*	*d.*	
Multiply	7	12	6	by 38.

	£.	*s.*	*d.*
Since $6 \times 6 = 36$, and $36 + 2 = 38$, therefore	7	12	6
			6
The product of $6 =$	45	15	
		6	
The product of $6 \times 6 = 36 =$	274	10	
The product of 7 *l.* 12 *s.* 6 *d.* by $2 =$	15	5	
The product of $38 =$ £.	289	15	

	lb.	*oz.*	*dwt.*	*gr.*	
Multiply	7	5	17	20	by 68.

Becauſe $10 \times 7 = 70$, and

	lb.	*oz.*	*dwt.*	*gr.*		
$70 - 2 = 68$, therefore	7	5	17	20		
				10		
	74	10	18	8		
				7		
	524	4	8	8	= the product of	70.
	14	11	15	16	= the product of	2.
lb.	509	4	12	16	difference =	68.

39. Where

39. When the multiplier exceeds 100, the moſt eligible way is that below, which will appear evident by inſpection.

EXAMPLE.

	s. d.			
Multiply	19 $4\frac{1}{2}$ by 126.			
	10			
	9 13 9			
	10			
	96 17 6	=	the product of	100
£. 9 13 9 × 2 =	19 7 6	=	the product of	20
19 $4\frac{1}{2}$ × 6 =	5 16 3	=	the product of	6
	£. 122 1 3	=	the product of	126

	s. d.			
Multiply	9 $4\frac{1}{2}$ by 396.			
	s. d.			
	9 $4\frac{1}{2}$ × 6			
	10			
	4 13 9 × 9			
	10			
	46 17 6			
	3			
	140 12 6	=	the product of	300
	42 3 9	=	the product of	90
	2 16 3	=	the product of	6
	£. 185 12 6	=	the product of	396

40. Becauſe this rule extends to any proportion where the diviſor is an unit, and conſequently extremely

tremely useful in calculating the value of any quantity of goods not exceeding a thousand *, we shall add a few practical questions.

What is the value of 43 *lb.* of indigo at 6 *s.* 4 *d. per lb.* Ans. 13 *l.* 12 *s.* 4 *d.*

97 *Cwt.* of cheese at 1 *l.* 5 *s.* 3 *d. per cwt.* Answer 122 *l.* 9 *s.* 3 *d.*

17 Yards of holland at 7 *s.* $8\frac{1}{2}$ *d. per* yard. Answer 6 *l.* 11 *s.* $0\frac{1}{2}$ *d.*

74 Yards of stuff, at 1 *s.* $4\frac{1}{2}$ *d. per* yard. Answer 5 *l.* 1 *s.* 9 *d.*

$6\frac{1}{2}$ Dozen pair of mitts, at 1 *s.* 10 *d. per* pair. Answer 7 *l.* 3 *s.*

63 Gallon of brandy, at 12 *s.* 4 *d. per* gallon. Answer 38 *l.* 17 *s.*

84 Pieces of broad cloath, at 16 *s.* 9 *d. per* yard. Ans. 70 *l.* 7 *s.*

7 Dozen of hats at 10 *s.* 8 *d.* each. Answer 44 *l.* 16 *s.*

$7\frac{1}{2}$ Dozen of plated buttons, at 3 *s.* 4 *d. per* dozen. Answer 1 *l.* 5 *s.*

79 *lb.* of tea, at 9 *s.* 4 *d. per lb.* Ans. 36 *l.* 17 *s.* 8 *d.*

396 *lb.* of tobacco, at 1 *s.* 2 *d. per lb.* Ans. 23 *l.* 12 *s.*

56 Ounces of silver, at 6 *s.* 10 *d. per* ounce. Answer 155 *l.* 16 *s.*

135 Gallons of rum at 7 *s.* 5 *d. per* gallon. Answer 50 *l.* 1 *s.* 3 *d.*

336 Packs of yarn, at 8 *l.* 16 *s.* 5 *d. per* pack. Answer 2963 *l.* 16 *s.*

729 Hogsheads of tobacco, at 3 *l.* 7 *s.* $9\frac{1}{2}$ *d. per* hhd. Answer 2471 *l.* 0 *s.* $1\frac{1}{2}$ *d.*

41. I shall next give somes questions in what is called *continued multiplication.*

What number is that, which being divided continually by 1, 2, 3, 4, 5, 6, 7, 8, 9, shall leave no remainder?

* When the quantity is large, the work is better and easier performed by the Rule of three or Practice. We may also observe, that when the quantity contains $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, those parts must be taken off the price and added to the product.

The truth of this example is easily examined, by dividing by 9, 8, 7, &c.

$$\begin{array}{r} 1 \\ 2 \\ \hline 2 \\ 3 \\ \hline 6 \\ 4 \\ \hline 24 \\ 5 \\ \hline 120 \\ 6 \\ \hline 720 \\ 7 \\ \hline 5040 \\ 8 \\ \hline 40320 \\ 9 \\ \hline 362880 \end{array}$$

answer.

How many changes may be rung on 6 bells?

Here $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ changes the answer.

A roper gave his daughter 20 ropes for her portion, every rope had 20 knots, at every knot hung 20 purses, in every purse were 20 pounds, I demand the Lady's fortune? $20 \times 20 \times 20 \times 20 = 160000$ *l.* answer.

A gentleman asked his host what he would take for his board so long as he could place himself and family, consisting of 7 persons, in a different position every day at dinner? The host, thinking it could not be long, said 5 *l.* to which the gentleman agreed. How long had he a right to stay? Answer $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$ days.*

DI-

* Which is almost 14 years. Many more examples of this kind might be proposed; but these are sufficient to shew the amazing power of numbers by continued multiplication.

DIVISION *of* COMPOUND QUANTITIES.

RULE.

42. *Divide the integral part by the rule for whole numbers (art. 27), then multiply the remainder by so many of the next inferior denomination as make* 1 *in that preceding it, taking in, or adding the quantity in the next lower denomination, (in the dividend) to the product, to which bring the divisor in order of division, and divide thereby; in like manner proceeding with the other remainders and denominations, setting the several quotients in order after each other for the answer.*

EXAMPLES.

Divide 4679 *l.* 12 *s.* 4 *d.* among 13 persons equally.

Divisor.	Dividend.			Quotient.			
	£.	s.	d.	£.	s.	d.	f.
13)	4679	12	4	(359	19	4	$3\frac{9}{13}$ *
	39						6
	77			2159	16	4	$\frac{1}{2}$
	65						2
	129			4319	12	9	
	117			359	19	4	$\frac{3}{4}$
	12						9 remainder.
	20			4679	12	4	proof.
	s.						
13)	252 (19						
	13						
	122						
	117						
	5						
	12						
	d.						
13)	64 (4						
	52						
	12						
	4						
	f.						
13)	48 (3						
	39						
	9						

* By consulting the rule and inspecting the work, the method will appear easy. We may observe, that the 9 farthings remaining cannot be divided among 13 persons; but in calculations, where it is necessary to preserve the remainders, they are set as in the quotient, and are called *fractions*, the value of which (in the present case) is nine-thirteenths of a farthing to each man.

Divide

Divide 7463 *oz.* 19 *dwt.* 19 *gr.* by 376.

	oz. dwt. gr.	*oz. dwt.*
376)	7463 19 19	(19 17 answer.
	376	
	3703	
	3384	
	319	
	20	
376)	6399	(17
	376	
	2639	
	2632	
	gr.	
dwt.	7 19 remainder.	

Proof:

oz.	*dwt.*		
19	17	× 6	
	10		
198	10	× 7	
	10		
1985			
3			
5955			
1389	10		
119	2		
	7	19	remainder.
Oz. 7463	19	19	proof.

Divide 749 *cwt.* 1 *qr.* 14 *lb.* among 1349 seamen equally.

	cwt. qr. lb.	
1349)	749 1 14	(
	4	
1349)	2997	(2
	2698	
	299	
	28	
	2392	
	598	
	14	
	8386	(6
	8094	
	292 remain.	

cwt.	*qr.*	*lb.*	
0	2	6	answer.
		10	× 9
5	2	4	× 4
		10	
55	1	12	× 3
		10	
553	2	8	
166	0	8	
22	0	16	
4	3	26	
2	2	12	remainder.
Cwt. 749	1	14	

Divide

Divide 74964 *l.* 13 *s.* $6\frac{1}{2}$ *d.* among 400 ſeamen, including 8 officers, to have ſhares as follows: The firſt 8 common men's ſhares, the ſecond 7, the third 4, the fourth 5, and the other 4 each 3. What are the reſpective ſhares of the officers, and what is a common man's ſhare?

From	400	ſeamen
Take	8	officers
Diff.	392	com. men

Then add:

392	common men's ſhares
8	dit. for the 1ſt officer
7	dit. for the 2d dit.
6	dit. for the 3d dit.
5	dit. for the 4th dit.
12	dit. for the 4 rem. of.
430	ſing. ſhares the diviſ.

	£.	s.	d.	£.	s.	d.	
430)	74964	13	$6\frac{1}{2}$	(174	6	$8\frac{3}{4}$	a ſingle ſhare
	430					8	

Work	£.	s.	d.			
3196	1394	13	10	=	8	dit. for the 1ſt
3010	1220	7	$1\frac{1}{4}$	=	7	dit. for the 2d
	1046	0	$4\frac{1}{2}$	=	6	dit. for the 3d
1864	871	13	$7\frac{3}{4}$	=	5	dit. for the 4th
1720	523	0	$2\frac{1}{4}$	=	3	dit. for the 5th
	523	0	$2\frac{1}{4}$	=		dit. for the 6th
144	523	0	$2\frac{1}{4}$	=		dit. for the 7th
20	523	0	$2\frac{1}{4}$	=		dit. for the 8th
s.	68339	17	10	=	392	dit. com. men
430) 2893 (6						
2580	74964	13	$6\frac{1}{2}$	=	430	dit. the proof.

313
12
430) 3762 (8
3440
322
4
1290
1290

£.	s.	d.	
174	6	$8\frac{3}{4}$	× 2
		10	
1743	7	$3\frac{1}{2}$	× 9
		10	
17433	12	11	
		3	
52301	18	9	
15690	5	$7\frac{1}{2}$	
348	13	$5\frac{1}{2}$	

£. 68339 17 10 = 392 single shares.

43. The same contractions may be used in compound division as in integral Numbers.

EXAMPLES.

	£.	s.	d.	
Divide	7466	14	2	by 3.
3)	7466	14	2	
	2488	18	0	$2\frac{2}{3}$

ILLUSTRATION.

Having proceeded with the whole numbers as directed at art. 20, there remains 2 from pounds, which multiplied mentally by 20, taking in the 14 s. will be 54, and the 3s in 5 once and 2 over; but the 3s in 24 are 8 times and 0 over; then the 3s in 2 are no times and 2 over, which multiplied by 4 is 8, and the 3s in 8 are 2 times and 2 over, that is $2\frac{2}{3}$ farthings, hence the quotient is 2488 *l.* 18 *s.* 0 *d.* $2\frac{2}{3}$ *f.* The learner is carefully to observe, that what remains from pounds must be multiplied in the mind by 20, taking in, or adding the shillings

lings in the dividend to the product, which divided by the given divisor, the quotient is put down as shillings, and the remainder multiplied by 12, taking the pence in the dividend into the product, it is again divided with the divisor, &c. the like is to be understood of other denominations, which I would have the learner to be particularly careful in observing, having found, in the course of teaching, that youth are frequently mistaken in this respect, and therefore unable to bring their work to a true conclusion.

Divide 749 *l.* 12 *s.* 9 *d.* by 36.

	£.	s.	d.
6)	740	12	9
6)	124	18	$6\frac{1}{2}$
£.	20	16	$5\frac{1}{12}$

Divide 257 *l.* 2 *s.* 6 *d.* among 12 persons equally. Ans. 21 *l.* 8 *s.* 6 *d.*

The divisor 120, the dividend 154 *l.* 17 *s.* 6 *d.* what is the quotient? Answer 1 *l.* 5 *s.* $9\frac{3}{4}$ *d.*

If 845 oxen cost 10583 *l.* 12 *s.* 6 *d.* what is the price of one ox? Answer 12 *l.* 10 *s.* 6 *d.*

If 648 yards of satin cost 405 *l.* what's the price of one yard? Ans. 12 *s.* 6 *d.*

A trader cleared 1467 *l.* in 19 years, what is that a-year? Ans. 77 *l.* 4 *s.* 2 *d.* $2\frac{2}{19}$ *f.*

Divide 14460 *l.* prize-money among 320 sailors, and give 5 a double share. Ans. The 5 had each 88 *l.* 19 *s.* $8\frac{100}{325}$ *d.* the others each 44 *l.* 9 *s.* $10\frac{50}{325}$ *d.*

If a person spend 43 *l.* 12 *s.* in a month, or 30 days, what is that *per* day? Ans. 1 *l.* 9 *s.* 0 *d.* $3\frac{6}{30}$ *f.*

Suppose 25 oxen cost 292 *l.* 12 *s.* what will one ox cost? Ans. 11 *l.* 14 *s.* 0 *d.* $3\frac{21}{25}$ *f.*

Divide 549 *l.* by 72. Ans. 7 *l.* 12 *s.* 6 *d.**

R E-

* This example proves the 1st example in Multiplication, art. 37, and in like manner may any of the rest be proved.

REDUCTION.

DEFINITION.

44. REduction is the changing of one denomination to that of some other, but retaining the same value. If any given denomination is reduced to another, but superior, it is called *reduction ascending*; but, if to an inferior one, *descending*.

RULE.

If a superior denomination is to be reduced to an inferior, multiply continually by so many of the next inferior denomination as make one in that preceding it, going thus through all the intermediate denominations, taking in or adding the several quantities in each kind to their several products respectively as you proceed : but to reduce an inferior denomination to one superior, the very contrary method must be used, viz. *that of a continual division.*

EXAMPLES.

	£.	s.	d.	
In	746	14	$6\frac{3}{4}$	how many farthings.
	20			
	14934	shillings		
	12			
	179214	pence		
	4			
	716859	farthings.		

In

. In 716859 farthings, how many pounds?

```
  4) 716859
     ------
 12) 179214¾
     -------
 2|0) 1493|4 6
      --------
  £. 746 14 6¾*
```

In 764 *l.* 10 *s.* 8 *d.* how many crowns, half-crowns, shillings, pence, and halfpence, and of each an equal number.

```
5 0 a crown                   £.  s. d.
2 6 half-a-crown             764  10 8
1 0 a shilling                20
0 1 a penny                 ------
0 0½ a halfpenny             15290
----                            12
8 7½ sum                    ------
 12                          183488
----                             2
103                         ------
  2                  207) 366976 (1772 172/207 answer
----                      207...
207 halfpence             ----
                           1599
                           1449
                           ----
                            1507
                            1449
                            ----
                             586
                             414
                             ---
                             172
                             ---
```

How

* These two examples prove each other. The method of performing the operation is so easy, that an inspection of the work will be a sufficient illustration.

How many farthings are in 74646 *l.* Anſwer 71660160 farthings.

In 71660160 farthings, how many pounds? Anſwer 74646 *l.*

In 21 guineas how many farthings? Anſwer 21168 farthings.

In 21168 farthings, how many guineas? Anſwer 21 guineas.

In 1460 crowns, how many nobles at 6 *s.* 8 *d.* each? Anſwer 1095 nobles.

In 1095 nobles, how many crowns? Anſwer 1460 crowns.

In 72 *l.* 19 *s.* 6 *d.* how many halfpence? Anſwer 25028 halfpence.

In 25028 halfpence, how many pounds? Anſwer 72 *l.* 19 *s.* 6 *d.*

In 749 *l.* 12 *s.* 6 *d.* how many ſix-pences? Anſwer 29985 ſix-pences.

In 29985 ſix-pences, how many pounds? Anſwer 749 *l.* 12 *s.* 6 *d.*

In 74965 pieces of eight, each at 4 *s.* 6 *d.* how many pounds? Anſwer 16867 *l.* 2 *s.* 6 *d.*

In 16867 *l.* 2 *s.* 6 *d.* how many pieces of eight? Anſwer 74965 pieces.

RE-

REDUCTION *of* WEIGHTS.

Of TROY WEIGHT.

```
   lb.  oz. dwt. gr.
In 469  10   19   14 how many grains?
    12
  ----
  5638 ounces
    20
 ------
112779 pennyweights
    24
 ------
451120
225559
-------
2706710 grains.
-------
```

In 2706710 grains, how many pounds Troy?

```
   6) 2706710
      -------
    4) 451118 2
       ---------
  2|0) 11277|9 2
       ---------
   12) 5638  19
       ---------
lb. 469  10  19  14* answer.
```

In 769 ingots of silver, weighing, one with another, 3 *lb.* 11 *oz.* 18 *dwt.* 18 *gr.* *per* ingot, how many punch ladles, each 2 *oz.* 10 *dwt.* 10 *gr.* tea-spoons, each 7 *dwt.* 14 *gr.* table-spoons, each 2 *oz.* 14 *dwt.* 19 *gr.* tea-tongs, each 2 *oz.* 16 *dwt.* salvers, each 3 *oz.* 19 *dwt.* cups, each 4 *oz.* 10 *dwt.* pints, each 5 *oz.* 17 *dwt.* and quarts, 10 *oz.* 16 *dwt.* of each an equal number?

oz.

* The learner will find no difficulty in reducing these weights and measures, if he consults the tables, art. 33. The 14 grains are obtained as directed per note to art. 30.

oz.	dwt.	gr.		*lb.*	*oz.*	*dwt.*	*gr.*
				3	11	18	18
2	10	10		12			
	7	14		———			
2	14	19		47			
2	16			20			
3	19			———			
4	10			958			
5	17			24			
10	16			———			
———	———	———		3840			
33	10	19		1917			
20				———			
———				23010			
670				769			
24				———			
———				207090			
2689				138060			
1341				161070			
———				———			

16099) 17694690 (1099 answer
16099...
———
159569
144891
———
146780
144891
———
1889
———

In 74 *lb.* 10 *oz.* 12 *dwt.* 21 *gr.* how many grains? Anfwer 431349 grains.

In 431349 grains, how many pounds Troy? Anfwer 74 *lb.* 10 *oz.* 12 *dwt.* 21 *gr.*

In 172 crown-pieces, each 18 *dwt.* 14 *gr.* how many *lb.* Troy? Anfwer 13 *lb.* 3 *oz*, 16 *dwt.* 8 *gr.*

In 13 *lb.* 3 *oz.* 16 *dwt.* 8 *gr.* how many crown pieces, each 18 *dwt.* 14 *gr.* Anfwer 172 crowns.

Of

Of AVOIRDUPOIS WEIGHT.

In 64 *tons.* 14 *cwt.* 1 *qr.* 19 *lb.* 4 *oz.* 13 *dr.* how many Drams? Anſ. 37113677 *dr.*

In 37113677 drams how many tons? Anſ. 64 *tons* 14 *cwt.* 1 *qr.* 19 *lb.* 4 *oz.* 13 *dr.*

In 21 bags of pepper, each 3 *qr.* 12 *lb.* how many *Cwts*? Anſ. 18 *cwts.*

In 18 *cwt.* of Pepper how many bags, each 3 *qrs* 12 *lb.* Anſ. 21 bags.

46. Becauſe it frequently is wanted to reduce hundred weights to pounds, I ſhall give ſome examples performed different ways, and thoſe the moſt eaſy and practical.

In 73 *cwt.* 2 *qr.* 14 *lb.* how many pounds?

4	2 or thus,	3 thus,	4 thus	5 alſo thus,
	73	146	146	7370
294	73	73	803	876
28	73	7370	70	8246 *lb.* *
2356	7370	8246 *lb.*	8246 *lb.*	
589	8246 *lb.*			
8246 *lb.*				

In 46 *cwt.* 1 *qr.* 14 *lb.* how many pounds?

46	92	92	4642
46	46	506	552
4642	4642	42 odd wt.	5194 *lb.*
5194 *lb.*	5194 *lb.*	5194 *lb.*	

* In the firſt method we multiply by 4 and 28, becauſe 28 *lb.* is 1 quarter, and 4 quarters 1 *cwt.* in the ſecond we conſider 112 *lb.* as 1 *cwt.* and therefore place 73 down twice, inſtead of multiplying by 2, and then other twice removed each time 1 place more to the left hand, which is the ſame thing as multiplying by the two units or 11, and the 2 *qr.* 14 *lb.* being 70 *lb.* is placed below as the odd weight, and all added together; the third method is performed in the ſame manner as the 2d, only the firſt line is the double of the *cwts.*

In 462 *cwt.* 3 *dr.* 21 *lb.* how many pounds?

462	924	924	46200
462	462	5082	105
462	462	105	5564
105	105	51849	49869
51849 *lb.*	51849 *lb.*		

In 64 *cwt.* 3 *qr.* 14 *lb.* how many *lb*? Anſ. 7266 *lb.*
In 63 *cwt.* 2 *qr.* 19 *lb.* how many *lb*? Anſ. 7131 *lb.*

Of LONG-MEASURE.

47 In 7626 *miles*, 7 *f.* 19 *p.* $4\frac{1}{2}$ *yd.* 2 *f.* 10 *i.* 2 *b.* how many barley-corns? Anſ. 1449728276.

In 1449728276 barley-corns, how many miles? An. 7626 *miles*, 7 *f.* 19 *p.* $4\frac{1}{2}$ *yd.* 2 *f.* 10 *i.* 2 *b.*

How many barley-corns will reach round the globe of our earth, being 360 degrees, each $69\frac{1}{2}$ miles? Anſ. 4755801600.

How many times will the large wheel of the Kendal ſtage coach, being $6\frac{3}{4}$ yards circumference, turn over between London and Kendal, diſtant 257 miles? Anſ. $67010\frac{10}{27}$ times.

It will be unneceſſary to multiply examples under all the tables.

Of the RATIO *of* NUMBERS.

48 The ratio of numbers conſiſts in their compariſon or relation in reſpect of magnitude, numbers increaſing or decreaſing by the continual addition or ſubtraction of ſome common quantity called the ratio, are ſaid

Inſtead of placing them twice down; the fourth method is the *cwts.* doubled in the firſt, and in the ſecond the ſaid *cwts.* multiplied by 11, and removed one place towards the left, the third line is the 70 *lb.* odd weight, the ſum of which is alſo the anſwer; and in the fifth method the *cwts* are multiplied by 100, and the odd weight taken in, the 2d line is the product of the *cwts.* by 12, the ſum being alſo the anſwer.

ſaid to be in arithmetical progreſſion, thus, 1, 3, 5, 7, 9, 11 and 12, 10, 8, 6, 4, 2, are each a ſeries of numbers in *arithmetical progreſſion*; the firſt increaſing by the continual addition of two, and the other decreaſing by the continual ſubtraction of 2.

49. And the numbers 2, 4, 8, 16, 32, 64 and 96, 48, 24, 12, 6, 3, are two ſeries of numbers, the firſt increaſing by the continual multiplication of 2 called the ratio, and the latter by the continual diviſion of 2; theſe, we ſay, are in *geometrical progreſſion*.

50. Four numbers are ſaid to be proportional, alſo if the ratio between the firſt and ſecond be the ſame as that between the third and fourth, thus, 4 : 2 : : 6 : 3, here 4 contains 2 as many times as 6 does 3, and the contrary; and hence it is that the product of the extremes are equal, the product of the Means (the firſt and fourth terms are called *extremes* with reſpect to the ſecond and third, which are the *Means*); for $4 \times 3 = 2 \times 6$, in which caſe the proportion is called *direct*. But four numbers are likewiſe proportional, if the product of the firſt and ſecond terms is equal, the product of the third and fourth thus; 16 : 2 : : 8 : 4, that is $16 \times 2 = 8 \times 4$, and this is termed *reciprocal* or *inverſe proportion*, becauſe the fourth term is to the third inverſely, as the ſecond is to the firſt.

RULE of THREE DIRECT;

Or GOLDEN RULE.

51. THE rule we now propoſe to treat of, is diſtinguiſhed with the appellation of *Golden Rule*, becauſe of its extenſive uſe in arithmetic. Nothing relative to computations of any kind, can be propoſed foreign to proportion.

DEFINITION.

In this rule there are always given three numbers to find a fourth, bearing the ſame proportion to the third

third, as the ſecond doth to the firſt; and ſince the product of the extremes is equal the product of the means, (*per* art. 50.) we have the following general rule.

RULE.

52. *State the queſtion by placing the three given numbers in ſuch order, that the firſt and third be of one kind, and the ſecond of the ſame kind as the fourth (or required anſwer); then multiply the ſecond and third terms together, and divide the product by the firſt, and the quotient will be the anſwer of the ſame denomination as the ſecond number. If the firſt and third terms be of different denominations, they muſt be reduced to the ſame, and the ſecond to the loweſt denomination mentioned, or lower if neceſſary.*

EXAMPLES.

If 4 yards of broad cloath coſt 3 *l.* what will 12 yards coſt?

Yds.	*£.*	*Yds.*
4 :	3 ::	12
		3
	4)	36
		9 *l.* anſwer.

If 1 *lb.* of tea coſt 12 *s.* what will 124 *lb.* coſt?

lb.	*s.*	*lb.*
1 :	12 ::	124
		12
	2|0)	148|8*
		£. 74 8 *s.* anſ.

Admit

* Becauſe the ſecond number is ſhillings, the anſwer is alſo in ſhillings, and muſt therefore be divided by 20 to reduce it to pounds.

Admit the Moon describes the ecliptic being 360 degrees in $27\frac{1}{2}$ days, how many degrees does he pass over in 1 day?

Days. ° *Days.*
$27\frac{1}{2}$: 360 : : 1
2 2 2
— —— —
55 5) 720 2
——
11 (144
——
Degrees $13\frac{1}{11}$ answer.

If a *cwt.* of sugar cost 3 *l.* 12 *s.* 6 *d.* what will 7 *cwt.* 3 *qr.* 14 *lb.* cost?

lb. £. *s.* *d.* *cwt.* *qr.* *lb.*
112 : 3 12 6 : : 7 3 14
20
—
72
12
——
870

798
84
——
882 *lb.*
870
——
61740
6956
——
112) 757340 (6761
672...
——
853
784
——
694
672
——
220
112
——
108

12
(6761
——
2|0) 56|3 5
——
£. 28 3 5 answer.
——

Laid out 191 *l.* 13 *s.* 4 *d.* for indigo, at 7 *s.* 8 *d.* *per lb.* now, for every 3 *lb.* of indigo, I bought 5 yards of broad cloth at 16 *s.* 6 *d.* *per* yard, how many *lb.* of indigo and yards of broad cloth did I buy? how must the indigo be sold *per lb.* and the broad cloth *per* yard, to gain 24 *l. per cent.* and what was gained by the whole?

```
s.  d.    lb.      £.    s.  d.
7   8  :  1  ::   191   13   4.
12                 20
--               ----
92               3833
                   12
                -----
          92) 46000 (500 lb.
              460..
              -----
              ...00

                 £.
                100
                 24
    £.          ---        s. d.
   100   :      124   ::   7  8
    20           92        12
  ----          ---        --
  2000          248        92
    12        1116
  -----       -----  £. s. d.
 24000 )      11408 (0  9  6 per yd.
                 20
             ------
   24000)    228160 (9
             216000
             ------
              12160
                 12
             ------
   24000)    145920 (6
             144000
             ------
               1920
             ------
```

lb.

lb. *yd.*
3 : 5 : : 500
5

3) 2500

833$\frac{1}{3}$ yards.

£. £. *s.* *d.*
100 : 124 : : 16 6
12

198
124

792
396
198

12) 245|52

20) 2|0 5

£. 1 0 5$\frac{1}{2}$

yds. *s.* *d.*
1 : 16 : : 833$\frac{1}{3}$
3 12 3

3 198 2500
2500

99000
396

3) 495000

12) 165000

2|0) 1375|0

£. 687 10

£. *s.* *d.*
191 13 4
687 10 0

£. £.
100 : 24 : : 879 3 4
6

5275
4

£. 211|00

Anſwer 500 *lb.* of indigo, 8333$\frac{1}{3}$ yards of broad cloath, 9 *s.* 6 *d.* *per lb.* received for the indigo, 1 *l.* 0 *s.* 5$\frac{1}{2}$ *d.* *received* for the broad cloath, 211 *l.* gained at the whole.

If 12 yards of cloth coſt 1 *l.* 16 *s.* what will 456 yards coſt? Anſwer 68 *l.* 8.

If 1 *lb.* of ſilver is worth 3 *l.* 2 *s.* what will 480 *oz.* 15 *dwt.* coſt? Anſ. 62 *l.* 3 *s.* 10$\frac{1}{2}$ *d.*

If

If 4 hogsheads of wine cost 56 *l.* 14 *s.* what will the value of one gallon be? Ans. 0 *l.* 4 *s.* 6 *d.*

What's the worth of 19 *oz.* 3 *dwts.* 5 *grs.* of gold at 2 *l.* 19 *s.* 4 *d.* *per* ounce? Ans. 56 *l.* 16 *s.* $10\frac{1}{4}$ *d.*

A merchant at London buys 64 tons of wine for 460 *l.* the freight cost 220 *l.* the loading, unloading, &c. 10 *l.* the customs 15 *l.* the charge of the cellar 8 *l.* and he would gain 250 *l.* A gentleman demands the value of 24 tons, what must he give? Answer 361 *l.* 2 *s.* 6 *d.*

If a yard of cloth cost 7 *s.* 6 *d.* what will 7 packs, containing 8 parcels and every parcel 108 yards, cost? Ans. 2268 *l.*

A grocer buys 24 *tons* 12 *cwt.* 2 *qr.* 14 *lb.* 12 *oz.* of tobacco for 3678 *l.* 6 *s.* 4 *d.* what will one ounce cost him? Ans. 0 *l.* 0 *s.* 1 *d.*

A boy being asked the time of the day, said, 'tis between 5 and 6 o'clock, and the hour hand and minute hand are together, what was the exact time of the day? Ans. $27\frac{3}{11}$ past 5 o'clock.

A was robbed by B, who flies 40 miles *per* day, but 3 days after A has intelligence of B, and pursues him at the rate of 50 miles *per* day; in how many days and after how many miles travel will A overtake B? Ans. in 12 days and 600 miles.

Lent 165 *l.* at 5 *l.* *per cent. per annum*, what will the interest be for 5 years? Ans. 41 *l.* 5 *s.* 0 *d.*

Suppose I spend 1 *s.* 9 *d.* *per* day, what is that in a year? Ans. 31 *l.* 18 *s.* 9 *d.*

Suppose I buy goods at 8 *s.* 6 *d.* and sell them for 13 *s.* 6 *d.* what do I gain *per cent.*? An. 58 *l.* 16 *s.* $5\frac{1}{2}$ *d.*

53. A great number of questions in this rule will admit of being much contracted, as will appear most conspicuous by examples.

Suppose 5 *lb.* of tea cost 1 *l.* 12 *s.* $6\frac{1}{2}$ *d.* what will 30 *lb.* require?

lb.	£.	*s.*	*d.*	*lb.*
5 :	1	12	$6\frac{1}{2}$ ::	30*
			6	
£.	9	15	3	Answer.

* Because 30 is 6 times 5, it will require 6 times so much as doth; therefore the second number multiplied by 6 gives the answer

If 1 yard coſt 7*s.* 4*d.* what will 16 yards coſt?

	s.	*d.*
	7	4
		4
1	9	4
		4
£. 5	17	4 Anſwer.

Suppoſe 25 yards coſt 22*l.* 10*s.* what will 8 yards coſt?

8

5) 180

5) 36

£. 7 4 0

Suppoſe 6*lb.* of indigo coſt 1*l.* 17*s.* 6*d.* what will 64*lb.* coſt?

	lb.	£.	*s.*	*d.*	*lb.*
	6 :	1	17	6 ::	64
Or,	3 :	1	17	6 ::	32
				8	

15

4

3) 60

£. 20 anſwer.

Suppoſe

Suppose 5 *lb.* of beef cost 1 *s.* $2\frac{1}{2}$ *d.* what will 125 *lb.* cost?

lb.		*s.*	*d.*		*lb.*
5	:	1	$2\frac{1}{2}$	: :	125
			5		
		6	$0\frac{1}{2}$		
			5		
£. 1		10	$2\frac{1}{2}$ answer.		

If 100 *l.* gain 5 *l.* what will 46 *l.* 10 *s.* gain?

	£.		£.		£.	*s.*
	100	:	5	: :	46	10
Or,					£.	*s.*
	20	:	1	: :	46	10
	2|0)		4|6		10	
			£. 2		6	6 answer.

The RULE *of* THREE INVERSE.

DEFINITION.

54. IN this rule there are given three numbers to find a fourth in such reciprocal proportion to the third as the second has to the first, agreeable to the latter part of art. 50.

RULE.

State the question, and reduce (when necessary) as in the direct rule; then multiply the second and first numbers together, and divide by the third, the quotient will

*will exhibit the anſwer in the ſame denomination as the ſecond number**.

EXAMPLES.

If 8 men, in 4 days, do any piece of work, how ſoon will 16 men do the ſame.

```
     m.  d.  m.
     8 : 4 :: 16
         --
16)  32 (2 d. Anſwer.†
     32
     --
     ..
```

If a perſon perform a journey in 12 days when the days are 16 hours long, how many days will it take him when the days are only 10 hours long.

```
        h.    d.    h.
        16 :  12 :: 10
        12
      ----
10)   192 (19 days. Anſwer
      10
      ---
       92
       90
       --
        2
```

If

* That you miſtake not queſtions in this rule, for thoſe in the direct, obſerve that the leſs the third number is, it will require a greater anſwer, and the contrary.

† Nothing can be more certain and eaſy to conceive than this, viz. That in the above example 16 men will perform the work in half the time that 8 perſons can do it in; the more hands will always, in ſimilar caſes, perform the buſineſs in leſs time; but in the direct rule the caſe alters, for if 1 yard require or coſt 1 *l.* then will 20 yards coſt 20 *l.* and a greater quantity will always coſt a greater ſum, from which obſervations the learner will be able to judge when his queſtion is inverſe, and when direct.

If a penny loaf weigh 8 ounces when the peck is worth 2*s.* 0*d.* what will a penny loaf weigh when the peck is valued at 1*s.* 6*d.* Anſ. 10*oz.* 13*dwt.* 8*gr.*

Borrowed 372*l.* for 7*yrs.* 8*mo.* how long muſt 496*l.* be lent to make an equivalent. Anſ. 5*yrs.* 9*mo.*

An Army beſieging a town in which were 1000 perſons with proviſions for 3 months, how many were diſcharged when the ſaid proviſions laſted 12 months. Anſ. 750 diſcharged.

If a room be 4 yards high, and 25 yards circumference, how much paper, 3 quarters wide, will paper the ſaid room. Anſ. $133\frac{1}{3}$ yards.

RULE *of* COMPOUND PROPORTION.

DIFINITION.

56. BY this rule may any compound proportional quantity be ſolved, if there are an odd number of terms given: I ſhall not here make any diſtinction between direct and inverſe proportion; but proceed to a general method which extends to all arithmetical computations where proportion is concerned. William Johns, Eſq; Fellow of the Royal Society, reduced it to the following form about 50 years ago, and it has been generally practiſed by ſucceeding mathematicians.

RULE.

57. *Let the terms expreſſing the conditions of the queſtion be put down in one line, under each of theſe ſet its correſponding one, in another line,* i. e. *place time under time, diſtance under diſtance, weight under weight, &c. then multiply the cauſes * in one line into the effects, in the other line for a dividend, obſerving to begin with that line which will give one term more in the dividend, than there will*

* By cauſes I mean ſomething capable of producing ſomething, which I call the effects of theſe cauſes, as men in time would produce work, which men and time are cauſes producing the effect, viz. Work; things carried, and the diſtance they are carried, produce wages; principal lent and time, are cauſes which produce intereſt, &c.

will the quotient be the answer.

EXAMPLES.

If 100*l.* in 12 months gain 5*l.* what will 75*l.* gain in 9 months.

	l. *m.* *l*		
	100 : 12 : 5		
100	75 : 9		
12	9		
1200	675	*l.* *s.* *d.*	
	5	2 16 3	answer.
12|00)	33|75		
	£. 2 975		
	20		
12|00)	195|00		
	S. 16 3 *d.*		

58. Now it is plain from the question that 100*l.* and 12 months are the causes which produce the effect 5*l.* I therefore, according to the rule, multiply the causes 75 and 9 by the effect 5 for a dividend and the remaining terms 100 into 12 is the divisor, the quotient being 2*l.* 16*s.* 3*d.* the required interest.

59. The method of performing questions of this kind is elegantly expressed as below.

$$\begin{array}{l} \text{Div.} \\ \text{Divis.} \end{array} \frac{75\times9\times5}{12\times100} = \frac{75\times9}{12\times20} = \frac{15\times9}{12\times4} = \frac{5\times9}{4\times4} = \frac{45}{16} =$$

2*l.* 16*s.* 3*d.* the Answer the same as before.

Now, if at any time there are the same quantities in the Divisor and Dividend, they may be left out; or if any number in either factor will divide some quantity in the other factor without a remainder, the quotient may be inserted instead of those quantities; or if any number

 will

will exactly meaſure two quantities, (viz one in each factor) the quotients of each, may be written for the quantities themſelves, to illuſtrate what has been ſaid let us reſume the quantity, $\frac{75 \times 9 \times 5}{12 \times 100}$ where it will eaſily appear that 5 will divide 100 and quote 20, therefore we write the quantity over again, leaving out the 5 and inſerting 20 inſtead of 100 thus $\frac{75 \times 9}{12 \times 20}$ now from conſulting this laſt expreſſion I find 5 will divide 75 and 20; therefore I write down the quotients inſtead of the numbers themſelves, and hence we have $\frac{15 \times 9}{12 \times 4}$; again ſeeing 3 will divide 15 and 12 we ſhall have $\frac{5 \times 9}{4 \times 4}$; but as no number will divide a quantity in each factor in the laſt expreſſion, it is incapable of any further contraction, we therefore multiply the terms of the factors, which become $\frac{45}{16}$ or $\frac{45}{4 \times 4}$ = 2*l.* 16*s.* 3*d.* The inſpection of a few examples, will make what is here advanced better underſtood.

If a carrier receives 2*l.* 2*s.* for the carriage of 3 *cwt.* 150 miles, how much ought he to receive for the carriage of 7 *cwt.* 3 *qr.* 19 *lb.* for 50 miles?—2*l.* 2*s.* is 42*s.* 7 *cwt.* 3 *qr.* 19 *lb.* is 882 *lb.* and 3 *cwt.* is 336 *lb.*

Therefore	*s.*	*lb.*	*m.*
	42 :	336 :	150
		882 :	50

Per Rule $\frac{42 \times 882 \times 50}{336 \times 150} = \frac{42 \times 882}{336 \times 3} = \frac{14 \times 882}{336} =$

$\frac{2 \times 882}{48} = \frac{441}{12} = \frac{147}{4} =$ 36*s.* 9*d.* = 1*l.* 16*s.* 9*d.* Anſ.

If 12 men build a wall 30 feet long, 6 feet high, and 3 feet thick, in 15 days; in how many days will 60 men make

make a wall 300 feet long, 8 feet high, and 6 feet thick?

m. *f.l.* *f.h.* *f.t.* *d.*

$$\left.\begin{array}{l}12 : 30 : 6 : 3 : 15 \\ 16 : 300 : 8 : 6 :\end{array}\right\}\text{Here } \frac{12\times15\times300\times8\times6}{60\times30\times6\times3} =$$

$$\frac{12\times15\times300\times8}{60\times30\times3} = \frac{15\times300\times8}{5\times30\times3} = \frac{3\times300\times8}{30\times3} =$$

$$\frac{300\times8}{30} = 10\times8 = 80 \text{ days, anſwer.}$$

A draper bought 27 pieces of cloth, each $24\frac{1}{2}$ yards long, and 7 quarters wide, at 14*s.* 8*d.* per yard; how many pieces of ſuch Cloth, each $31\frac{1}{4}$ yards long, and 5 quarters wide may be bought for 1375 *l.*

Y. *s.* *d.* *Yd.* *Pence*

$$\text{Firſt } 1 : 14\ 8 :: 24\tfrac{1}{2}\times27 : \frac{176\times49\times27}{2} = 88\times49\times27$$

$$\begin{array}{ccc} \frac{2}{2} & \underline{12} & \underline{2} \\ & 176 & 49 \end{array}$$

P. *Yds.* *Qur.* *Pence*

$$\text{Then } \left.\begin{array}{l}27 : 24\tfrac{1}{2} : 7 : 88\times49\times27 \\ \quad\ 31\tfrac{1}{4} : 5 : \quad 1375\,l.\end{array}\right\} \text{Now } 24\tfrac{1}{2} \text{ is } 98$$

quarters, $31\frac{1}{4}$ yards is 125 quarters, and 1375 *l.* is 330000 pence; therefore $\frac{27\times98\times7\times330000}{125\times5\times88\times49\times27} = \frac{98\times7\times330000}{125\times5\times88\times49}$

$$= \frac{2\times7\times330000}{125\times5\times88} = \frac{2\times7\times66000}{125\times88} = \frac{7\times66000}{125\times44} =$$

$$\frac{7\times13200}{25\times44} = \frac{7\times2640}{5\times44} = \frac{7\times528}{44} = \frac{7\times132}{11} =$$

$7\times12 = 84$ pieces the Anſwer.

Lent my friend 250 *l.* for 9 months, when the rate per cent. was 6 *l.* how long can I borrow 75 *l.* of him, when intereſt is at 5 *l.* per cent.

£. £.

$$\left.\begin{array}{l}1 : 250 : 6 : 9 \\ 1 : \ 75 : 5 :\end{array}\right\} \frac{250\times6\times9}{75\times5} = \frac{10\times6\times9}{3\times5} =$$

$$\frac{2\times6\times9}{3} = 2\times2\times9 = 36 \text{ months.}$$

If

If 16 men can build a wall 50 feet long, 7 feet high, and 3 feet thick, in 18 days, when the days are 12 hours long; in how many days will 48 men build 200 feet long, 9 feet high, and 4 feet thick, when the days are but 8 hours long.

m.	*f.l.*	*f.h.*	*f.t.*	*d.*	*h.*
16 :	50 :	7 :	3 :	18 :	12
48 :	200 :	9 :	4 :	:	8

$$\frac{16 \times 18 \times 12 \times 200 \times 9 \times 4}{48 \times 8 \times 50 \times 7 \times 3} = \frac{18 \times 12 \times 200 \times 9 \times 4}{3 \times 8 \times 50 \times 7 \times 3} = \frac{18 \times 12 \times 4 \times 9 \times 4}{3 \times 8 \times 7 \times 3} = \frac{18 \times 12 \times 4 \times 4}{8 \times 7} = \frac{18 \times 12 \times 4}{2 \times 7} = \frac{9 \times 12 \times 4}{7} =$$

$$\frac{432}{7} = 61\tfrac{1}{7} \text{ Answer.}$$

PRACTICE.

DEFINITION.

60. BY practice is underſtood, a compendious method of performing the rule of three direct, where the firſt number is an unit, or can eaſily be reduced thereto.

61. If the learner cannot immediately determine what money is the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of a pound, a ſhilling, &c. the following tables will aſſiſt him therein.

TABLES *of* ALIQUOT PARTS.*

f.			
1	is	$\frac{1}{48}$	Of a Shilling.
2	—	$\frac{1}{24}$	
d. 3	—	$\frac{1}{16}$	
1	—	$\frac{1}{12}$	
$1\frac{1}{2}$	—	$\frac{1}{8}$	
2	—	$\frac{1}{6}$	
3	—	$\frac{1}{4}$	
4	—	$\frac{1}{3}$	
6	—	$\frac{1}{2}$	

	d.			
	$1\frac{1}{2}$	is	$\frac{1}{160}$	Of a Pound.
	3	—	$\frac{1}{80}$	
	4	—	$\frac{1}{60}$	
	6	—	$\frac{1}{40}$	
s.	8	—	$\frac{1}{30}$	
1	0	—	$\frac{1}{20}$	
1	3	—	$\frac{1}{16}$	
1	4	—	$\frac{1}{15}$	
1	8	—	$\frac{1}{12}$	

2

* Aliquot parts are ſuch, that a certain number of them make up the whole, for 6 two-pences being a ſhilling, we ſay 2 *d.* is the $\frac{1}{6}$ of a ſhilling, &c.

s.	d.		
2		———	$\frac{1}{10}$
2	6	———	$\frac{1}{8}$
3	4	———	$\frac{1}{6}$
4		———	$\frac{1}{5}$
5		———	$\frac{1}{4}$
6	8	———	$\frac{1}{3}$
10		———	$\frac{1}{2}$

Of a Pound.

62. RULE I. *If the price be at a farthing, divide the quantity by* 4, 12, *and* 20. *If at a halfpenny by* 2, 12, *and* 20. *If at three farthings, multiply by* 3, *and divide by* 4, 12 *aud* 20, *and you have the respective anſwers.*

EXAMPLES.

To what comes 7496 yards, at 0¼*d.* per yard.

4) 7496
12) 1874
2|0) 15|6 2
£. 7 16 2 anſwer.

Required the value of 7698 *lb.* at 0½*d.* per *lb.*

2) 7698
12) 3849
2|0) 32|0 9
£. 16 0 9 anſwer.

What the value of 769 *lb.* at 0¾ *d.* per *lb.*

769
3
4) 2307
12) 576¾
2|0) 4|8 0
£. 2 8 0¾ anſwer.

7656 *lb.* at 0¾ *d.* per *lb.*

3
4) 22968
12) 5742
2|0) 47|8 6
£. 23 18 6 anſwer.

Queſtions.	Anſwers.	Queſtions.	Anſwers.
Yds. *d.*	*£.* *s.* *d.*	*Yds.* at *d.*	*£.* *s.* *d*
7649 at $0\frac{1}{4}$	7 19 $4\frac{1}{4}$	987 at $0\frac{1}{2}$	2 1 $1\frac{1}{2}$
4676 at $0\frac{1}{2}$	9 14 10	759 at $0\frac{3}{4}$	2 7 $5\frac{1}{2}$
7658 at $0\frac{3}{4}$	23 18 $7\frac{1}{2}$	6526 at $0\frac{1}{8}$	3 7 $11\frac{3}{4}$

RULE 2. *If the price be an aliquot, (or even) part of a pound, take that part of the given quantity, and it will be the anſwer.*

EXAMPLES.

7697 at 1 *s.*	76567 at 1 *s.* 8 *d.*	7962 at 2 *s.*
2|0) 769|7	12) 76567	10) 7962
£. 384 17	£. 6380 11 8	£. 796 4 *

Queſtions.	Anſwers.	Queſtions.	Anſwers.
Yds. *s.* *d.*	*£.* *s.* *d.*	*Yds.* *s.* *d.*	*£.* *s.* *d.*
749 at 10	374 10 0	746 at 4	149 4 0
467 at 6 8	155 13 4	976 at 3 4	162 13 4
678 at 5 0	169 10 0	976 at 2 6	122 00 0

RULE 3. *If the price conſiſts of an even number of ſhillings, multiply the quantity by half that number, doubling the units place for ſhilling, and the others will be pounds.*

EXAMPLES.

379 at 4 *s.*	3878 at 6 *s.*	7694 at 18 *s.*
2	3	9
£. 75 16 0	£. 1163 8	£. 6924 12

Queſtions.

* It appears that the units place is doubled for ſhillings, and that the other places of the quantity are pounds, and from this example it is, that the 3d rule is deduced.

Questions.		Answers.		Questions.		Answers.	
Yds.	s.	£.	s.	Yds.	s.	£.	s.
7695 at	2	769	10	756 at	8	604	16
849 at	4	169	16	796 at	10	398	0
721 at	6	432	12	879 at	12	527	8
721 at	14	504	14	769 at	16	615	4

65. RULE 4. *If the price consists of shillings, pence, &c. multiply by half the greatest number of even shillings, (per last article) and with the remainder of the price, take the aliquot part or parts of two shillings with what is then left, take the part of some of the foregoing parts, &c. 'till the whole price is exhausted, the sum total of the several lines thus obtained, is the answer.*

EXAMPLES.

To how much will 5762 *lb.* of Indigo amount at 6 *s.* 6 *d.* per *lb.*

	3	
6 *d.* is of 2 shillings the ¼)	1728	12
	144	1*
£.	1872	13

		7596 at 2 *s.* 8 *d.*	
		1	
8 *d.* is of 2 shillings the	⅓	759	12
		253	4
		1012	16
	£.		

8 *d.*

* When ever the parts are taken of 2 shillings, what remains from the place of tens prefixed before the units must be doubled, and the respective part thereof is shillings, what remains must be multiplied by 12, and divided by the part you take, &c.

		7694 at 8 *d.*
8 *d.* is of 2 ſhillings the	⅓	256 9 4
		8496 at 4 *s.* 8 *d.*
		2
8 *d.* is of 2 ſhillings the	⅓	1699 4
		283 4
	£.	1982 8
		7529 at 15 *s.* 8½ *d.*
		7
		5270 6
1 *s.* is of 2 *s.* the	½	376 9
8 *d.* is of 2 *s.* the	⅓	250 19 4
0½ *d.* is of 8 *d.* the	$\frac{1}{16}$	15 13 8½
	£.	5913 8 0½
6 *d* is of 2 ſhillings the	¼	3487 at 6¾ *d.*
¾ is of 6 *d.* the	⅛	87 3 6
		10 17 11¼
	£.	98 1 5¼

Queſtions.		Anſwers.		
Yds.	at *d.*	£.	*s.*	*d.*
1953	at 2¾	22	7	6¾
802	— 4¼	14	4	0½
370	— 4¾	7	6	5½
324	— 5	6	15	
3120	— 5½	71	10	
764	— 6	19	2	
674	— 8	22	9	4
3514	— 8¼	120	15	10½
5272	— 9	197	14	

Queſtions.

Questions.				Answers.		
Yds.	at	*s.*	*d.*	*£.*	*s.*	*d.*
2163	—	1	7	171	4	9
531	—	3	9	99	11	3
1734	—	6	$10\frac{1}{2}$	596	1	3
410	—	9	$7\frac{3}{4}$	197	14	$9\frac{1}{2}$
1209	—	12	7	760	13	3
612	—	16	$4\frac{3}{4}$	501	14	3
816	—	18	$2\frac{1}{4}$	742	1	0
970	—	19	8	953	16	8
575	—3*l.*	10	8*	2031	13	4

66. RULE 5. *When there are odd weights, &c. in the quantity, multiply the price by the integral part, (see articles 36, 37, 38, 39) and for the odd weight, take aliquot parts of the integer, &c. 'till the quantity is exhausted, taking those parts of the integral price, &c. the sum of which with the product of the integral quantity by the price, will be the answer.*

EXAMPLES.

Required the value of 7 *cwt.* 3 *qr.* 14 *lb.* at 1 *l.* 12 *s.* 6 *d* per *cwt.*

		£.	*s.*	*d.*
		1	12	6
				7
Qr. *Cwt.* 2 is of 1 the	$\frac{1}{2}$	11	7	6
			16	3
Qr. 1 is of 2 *qrs.* the	$\frac{1}{2}$		8	$1\frac{1}{2}$
14 *lb.* is of 1 *qr.* the	$\frac{1}{2}$		4	$0\frac{3}{4}$
	£.	12	15	$11\frac{1}{4}$

* If there are pounds in the given price, multiply the quant'ty by them, then proceed according to the rule.

What

What is the value of 64 *oz.* 10 *dwt.* 12 *grs.* at 6*s.* 3*d.* per ounce.

			8		
		2	10		
			8		
10 *dwt.* is of 1 *oz.* the	$\frac{1}{2}$	20			
12 *grs.* is of 10 *dwt.* the	$\frac{1}{20}$		3	$1\frac{1}{2}$	
				$1\frac{3}{4}$	$\frac{10}{20}$
	£.	20	3	$3\frac{1}{4}$ *	

Questions.				Answers.		
	£.	s.	d.	£.	s.	d.
17 *cwt.* 3 *qrs.* at	1	7	0	23	19	3
246 *cwt.* 3 *qrs.* 14 *lb.* at	5	15	$7\frac{3}{4}$	1427	9	$11\frac{1}{2}$
5 *oz.* 6 *dwt.* 17 *grs.* at		11	6	3	1	$4\frac{1}{2}$
32 *acres* 1 *rood* 14 *perches* at	1	16	0	58	4	$1\frac{3}{4}$
218 *cwt.* 3 *qrs.* 14 *lb.* at	5	15	$7\frac{3}{4}$	1265	11	$11\frac{3}{4}$

INTEREST.

DEFINITION.

67. INTEREST is a premium, or gratuity allowed to the lender, by the borrower, and is at so much per cent. per annum. The money lent is called the principal. The rate is the intereſt of 100*l.* for a year, the amount is the ſum of the principal and intereſt. Intereſt is likewiſe reſolved into ſimple and compound. Simple intereſt is for the principal alone, but compound intereſt † is not only allowed for the principal, but alſo for its intereſt, at the expiration of each ſtated payment.

Simple

* When the parts are many and complicated, it is much better to have recourſe to the rule of three direct.

† Compound intereſt will have a place in this work, after the rudiments of algebra.

SIMPLE INTEREST.

68. RULE. *Multiply the principal time and rate together, the product divided by* 100, *gives the interest which added to the principal gives the amount.**

EXAMPLES.

Required the intereſt 75 *l.* 10 *s.* at 5 *l.* per cent. per annum, for 1 year?

```
£.    £.    £.
100 : 5 : : 75  10
                 5
          ---------
          3|77  10
                20
          ---------
          15|50
             12
          ------
           6|00   £. 3 15 6 anſwer.
```

Or,

```
2|0) 7|5  10
-------------
£. 3 15 6† anſwer.
```

What is the amount of 562 *l.* 10 *s.* at 5 *l.* per cent. per annum, for 9 years?

```
         £.    s.
    2|0) 56|2  10
    -------------
£.    28  2  6
             9
    -------------
     253  2  6 intereſt.
     572 10  6 principal.
    -------------
£.   815 12  6 amount.
```

Re-

* If the time be ſome part or parts of a year, theſe muſt be taken inſtead of multiplying. If it be given in days, or is neceſſary to be reduced thereto, ſay as 365 days : 1 years intereſt : : the given days to the intereſt.

† Since 100 contains 5 twenty times, therefore when the rate is at 5 *l.* per cent. dividing the principal by 20, gives the intereſt for 1 year.

Required the intereſt 49 *l.* 10 *s.* at $4\frac{1}{2}$ *l.* per cent. per annum, for $7\frac{1}{2}$ years?

£. s.
49 10
$4\frac{1}{2}$

198
24 15 for the $\frac{1}{2}$ *l.*

---

222 15
$7\frac{1}{2}$

1559 5
111 7 6

16|70 12 6
20

14|32 £. s. d.
12 16 14 $1\frac{1}{2}$ anſ.

1|50
4

2|00

What is the intereſt of 46 *l.* at 4 *l.* per cent. per annum, for $\frac{3}{4}$ of a year)

46
4

184

---

6 months of 12 the $\frac{1}{2}$ 92
3 months of 6 the $\frac{1}{2}$ 46

---

1) 38
20

---

7) 60
12 1 *l.* 7 *s.* 7 *d.* anſwer.

---

7) 20

What

What is the amount of 1436 *l.* at $3\frac{1}{2}$ *l.* per cent. per annum, for 5 *y.* 3 *m.* 2 *w.* 3 *d.*

```
    £.
   1436
      3½
  ------
   4308
    718
  ------
50) 26
     20          40
  ------          4
 5) 20          ----
     12       1) 60
    ----        ----
  2) 40
```

Then as 365 *d.* : 50 *l.* 5 *s.* $2\frac{1}{4}$ *d.* : : 5 *y.* 3 *m.* 2 *w.* 3 *d.*

```
        20              13
     -------          -----
      1005              68
        12               4
     -------          -----
     12062             274
         4               7
     -------          -----
     48249            1921
      1921
     -------
     48249
    96498
   434241
   48249
   --------
365) 92686329
```

 365) 92686329

```
                                  4
365) 92686329           ( 253935
     730                  ------
     ----               12) 63483¾
     1968                -----------
     1825               2|0) 529|0 3
     -----                   ------
      1436                  264 10 3¾
      1095             £. 1436
      ----              -------------
      3413             £. 1700 10 3¾ answer
                        -------------
      3285
      -----
       1282
       1095
       -----
        1879
        1825
        -----
         54
```

What is the interest of 450 *l.* for a year, at 5 *l.* per cent. per annum? Answer 22 *l.* 10 *s.*

What is the interest of 720 *l.* for 3 years, at 5 *l.* per cent. per annum? Answer 108 *l.*

What is the interest of 284 *l.* 7 *s.* 6 *d.* for $1\frac{1}{2}$ years, at 5 *l.* per cent. per annum? Answer 21 *l.* 6 *s.* $6\frac{3}{4}$ *d.*

What is the interest of 248 *l.* 7 *s.* 6 *d.* for 3 years and 7 months, at $4\frac{1}{2}$ per cent. per annum?

Answer 40 *l.* 2 *s.* $1\frac{1}{2}$ *d.*

Required the commission of 729 *l.* 10 *s.* 6 *d.* worth of goods, at $2\frac{1}{8}$ per cent? Answer 15 *l.* 10 *s.* $\frac{1}{2}$ *d.*

What will a Brokers commission come to, upon 265 *l.* 10 *s.* worth of goods, at $3\frac{3}{4}$ per cent?

Answer 9 *l.* 19 *s.* $1\frac{1}{2}$ *d.*

Sent to my factor a quantity of goods, which per advice he sells for 472 *l.* but has paid carriage, freight, porterage, &c. 36 *l.* he charges ware-house room 1 *l.* 5 *s.* insurance from fire, at 5 *s.* per cent. risque of bad debts at 10 *s.* per cent. and 4 *l.* per cent. for 200 *l.* advanced on my account, his commission on the whole at

at $2\frac{1}{2}$ per cent, what remains due to me according to the conditions of this account? Anſwer 211 *l.* 8 *s.* 3 *d.*

What intereſt is due upon a bond of 634 *l.* 16 *s.* 4 *d.* at $4\frac{1}{2}$ per cent. per annum, from June 15th, 1768, to September 10th, 1770, (being 2 years and 87 days?)
Anſwer 63 *l.* 18 *s.* 10 *d.*

69. The method of calculating the commiſſion on goods, &c. being performed in the ſame manner as intereſt, we judge it unneceſſary to enter into any diſtinction, preſuming that a perſon who can find the intereſt of a ſum of money, will as eaſily find the commiſſion on Goods valued or ſold for ſuch a ſum, at any rate per cent. The reader will find intereſt treated upon in a more general and ſcientific manner, after the Eliments of Algebra.

DISCOUNT or REBATE.

DIFINITION.

70. *DISCOUNT* is the difference between a debt due ſome time hence and its preſent value.

The *preſent worth* or value of any debt due ſometime hence, is a ſum which put to intereſt for the given time and rate per cent. agreed upon, would amount to the given debt.

71. RULE. *As the amount of* 100 *l. for the given time and rate*
Is to 100 *l. or its intereſt;*
So is the given debt
To the preſent value, or diſcount reſpectively.

EXAMPLES.

What is the diſcount of 74 *l.* 10 *s.* at 5 *l.* per cent. per annum, for 6 months?

The intereſt of 100*l.* in 6 months at 5*l.* per cent. per annum, is 2*l.* 10*s.* and the amount thereof, 102*l.* 10*s.* therefore per Rule.

```
 £.  s.   £.     £.  s.
 102 10 : 100 :: 74 10
     20             20
 ------          -----
 2050              1490
                    100
                 ------   £.  s.  d.
          205|0)14900|0 (72  13   7 the preſent worth
                 1435
                 -----
                  550
                  410
                 -----
                  140
                   20
                -------
           205) 2800 (13
                 205
                -----
                 750
                 615
                -----
                 135
                  12
                -----
           205) 1620 (7
                1435
                -----
                 185
```

Again,

	£. s.		£. s.		£. s.
Again,	102 10	:	2 10	: :	74 10
	20		20		20
	2050		50		1490
					50

205|0) 7450|0 (3|6 — 2|0
615
1300
1230
70
12

205) 840 (4
820
20

1 16 4 the disc.

		£.	s.	d.	rem.
Hence,	The present worth	72	13	7	185
	The discount	1	16	4	20
	The sum is £.	74	10		

What is the present value of 700 *l.* at 5 *l.* per cent. per annum, due 9 months hence? Ans. £. 674 13 $11\frac{2}{4}$ $\frac{70}{81}$.

Bought goods to the amount of 83 *l.* 6 *s.* to pay 6 months hence, what present money will pay for them discount, at 8 *l.* per cent. per annum?

Answer, 80 *l.* 1 *s.* 11 *d.* $\frac{16}{108}$.

What is the present value of 326 *l.* 5 *l.* at 4 *l.* per cent. per annum, payable in $7\frac{1}{2}$ months?

Answer, 318 *l.* 5 *s.* 10 *d.*

What is the difference between the discount and interest of 49 *l.* 10 *s.* at $4\frac{1}{2}$ *l.* per cent. per annum, due $7\frac{1}{2}$ years hence? Answer, 4 *l.* 4 *s.* 5 *d.*

What present money will pay a debt of 120 *l.* payable as follows, viz. 50 *l.* at 3 months, 50 *l.* at 5 months, and the remainder at 8 months, discount at 6 *l.* per cent. per annum? Answer, 117 *l.* 5 *s.* $5\frac{1}{4}$ *d.*

Required the difference between the interest and discount of 500 *l.* due 12 months hence, at 5 *l.* per cent. per annum? Answer, 1 *l.* 3 *s.* $9\frac{3}{4}$ *d.*

Suppose a person buys goods which amount to 720 *l.* ready money, and another person buys the same quantity of such goods on 12 months credit for 750 *l.* whether has the better bargain; admit discount for present payment be at 8 *l.* per cent? Answer, the advantage is in favour of the latter, 25 *l.* 11 *s.* $1\frac{1}{3}$ *d.*

TARE and TRET.

DEFINITION.

72. GROSS is the weight of any commodity, together with the weight of the chest, box, bag, wrapper, &c.

Tare is an allowance made to the buyer for the weight of the chest, box, bag, &c.

Tret is an allowance made sometimes for waste, dust, &c. and is 4 *lb.* for 104 *lb.* or 1 *lb.* for 26.

Cloff is a further allowance of 2 *l.* for 3 *cwt.* to make the weight hold good when retail'd.

Neat is what remains, when these allowances are deducted.

EXAMPLES*.

Let the gross be 37 *cwt.* 2 *qr.* 14 *lb.* and the tare 4 *cwt.* 3 *qr.* 18 *lb.* what is the Neat?

	cwt.	qr.	lb.
From	37	2	14
Take	4	3	18
cwt.	32	2	24 neat.

In

* It will be to little purpose to give particular rules of the several cases which may happen, for two many directions only confound the learner, and the inspection of the examples will sufficiently illustrate the methods of operation.

In 5 bags of hops, weighing 15 *cwt.* 3 *qr.* 14 *lb.* how many *cwt.* neat, allowing tare at 10 *lb.* per bag?

5 bags
10 *lb.*
———
50 *lb.* = 1 *qr.* 22 *lb.* tare.

	cwt.	*qr.*	*lb.*	
From	15	3	14	grofs.
take		1	22	tare.
cwt.	15	1	20	

In 4 cafks of currants, each 7 *cwt.* 1 *qr.* 12 *lb.* grofs, the tare at 2 *qr.* 10 *lb* per cafk, tret 4 *lb.* per 104 *lb.* cloff 2 *lb.* per 3 *cwt.* how many *cwt.* neat?

	cwt.	*qr.*	*lb.*	
	7	1	12	
			4	
	29	1	20	gr.
	2	1	12	tare.
26)	27	0	8	tare futtle.
	1	0	$4\frac{16}{26}$	tret, being per queftion $\frac{1}{26}$ of the tare futtle.
	26	0	4	tret futtle.
			$17\frac{1}{3}$	cloff.
cwt.	25	3	15	neat.

	qr.	*lb.*	
	2	10	
		4	
cwt. 2	1	12	tare

cwt.	*qr.*	*lb.*
26	0	4
2		
3)52		
lb. $17\frac{1}{3}$ cloff.		

In 76 *cwt.* 2 *qr.* 14 *lb.* grofs, the tare at 21 *lb.* per *cwt.* how many *cwt.* neat?

	cwt.	*qr.*	*lb.*	
14 *lb.* is of 1 *cwt.* $\frac{1}{8}$)	76	2	14	
7 *lb.* is of 14 *lb.* the $\frac{1}{2}$)	9	2	8	
	4	3	4	
cwt.	14	1	12	tare.
cwt.	62	1	2	neat.

What

What other uſeful rules yet remain will be treated of after the doctrine of vulgar and decimal fractions, to which we proceed.

VULGAR FRACTIONS.

DEFINITION.

73. A *Fraction* is the part of ſome whole or integral quantity. If we ſuppoſe 1 *l.* to be divided into 20 equal parts, any number of them will be expreſſed by placing it above the 20, with a line between them: thus ſhould we want to expreſs 15 of thoſe parts, it is $\frac{15}{20}$ and reads *fifteen twentieths*, alſo $\frac{19}{20}$ is *ninteen twentieths*, but ſhould there be a quantity expreſſed by $\frac{20}{20}$ 'tis plain we have taken all its parts which are evidently equal to the whole or 1 *l.* again if we take $\frac{21}{20}$ the parts really exceed the whole. And hence fractions are naturally divided into two kinds.

1ſt, *A proper fraction* as, $\frac{19}{20}$ being leſs than the whole.
2d, *An improper fraction* as, $\frac{21}{20}$ which is greater than the whole.

74. But if the integer itſelf be a fraction, we expreſs them by the particle *of* between, and they are termed *compound fractions*, ſuch are $\frac{3}{4}$ of $\frac{5}{8}$, and $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{2}{3}$. But an expreſſion made up of a whole number, and a fraction is called *a mixed number* ſuch are $26\frac{1}{2}$ and $32\frac{3}{4}$. How theſe ſeveral kinds of fractions are fitted for addition, ſubtraction, &c. we proceed to determine.

REDUCTION *of* VULGAR FRACTIONS.

75. TO abreviate or reduce fractions to the loweſt terms.

RULE 1. *Divide both the numerator and denominator* by any number which will divide each, and leave no remainder,*

* The quantity ſtanding above the line is called the numerator, and ſhews how many parts you intend to expreſs by the fraction, and that below the denominator; and ſhews the number of parts contained in the integer.

*mainder, the quotients will be the numerator and denominator of a new fraction equal in value with the former *: this new fraction abbreviate again, &c. till you find nothing but unity will meaſure them both; the laſt fraction, thus diſcovered, is in the loweſt terms.*

RULE 2. *Divide the greater term of the fraction by the leſs, and the diviſor by the remainder, &c till 0 remain, the laſt diviſor thus diſcovered is called the greateſt common meaſure, which will abbreviate the given fraction, (by dividing the numerator and denominator) to the loweſt terms.*†

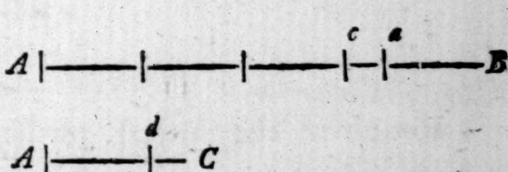
EXAMPLES.

Reduce $\frac{9}{12}$ to the loweſt terms.

$$\frac{9}{12} = \frac{3}{4} \text{ the anſwer.}$$

Reduce

* This if compared with what was ſaid concerning ſimiliar factors (art. 59) will be eaſily underſtood.



† Let *A B* (agreeable to the example) contain 44 and *A C* 12 parts, now the greateſt line which meaſures them both, cannot exceed *A C*: But dividing *A B* = 44 by *A C* = 12, the quotient is 3 times = *A a*, and 8 remains = *a B*. Now no number greater than *a B* can meaſure *A B* and *A C*; if it be denied, let us take *c B* greater than *a B*, and if it meaſures *A C* it muſt alſo meaſure *a A* = 3 times *A C* and *a B* alſo; but *B c* being greater than *B a* is ſuppoſing a part meaſured by the whole which is abſurd. Again let us try if *B a* will meaſure *A B* and *A C*, if it meaſures *A C* it will evidently meaſure *A B* (becauſe the line *A B* is equal 3 times *A C* added to *a B*), but *A C* divided by *a B*, i. e. 12 divided by 8, quotes 1 and 4 over which call *d C*, now by the foregoing method of reaſoning no line greater than *d C* can meaſure *A d* = *aB*; and by trial I find that *d C* = 4 will meaſure *B a* = *A d* = 8 for 8; divided by 4 quotes 2 and 0 over; and hence 'tis proved that *d C* is the greateſt line that will meaſure *A B* and *A C*, or 4 is the greateſt common meaſure of 44 and 12. *Q. E. D.*

Reduce $\frac{21}{63}$ to the loweſt terms.

$\frac{21}{63} = \frac{7}{21} = \frac{1}{3}$ the anſwer.

Reduce $\frac{192}{576}$ to the loweſt terms.

$\frac{192}{576} = \frac{96}{288} = \frac{48}{144} = \frac{24}{72} = \frac{12}{36} = \frac{6}{18} = \frac{3}{9} = \frac{1}{3}$ anſ.

Required the loweſt terms of $\frac{5184}{6912}$

Anſwer $\frac{3}{4}$

Reduce $\frac{52}{13}$ to the loweſt terms.

Anſwer $\frac{4}{1} = 4$

Reduce $\frac{96}{108}$ to the loweſt terms.

Anſwer $\frac{8}{9}$

Reduce $\frac{112}{120}$ to the loweſt terms.

Anſwer $\frac{14}{15}$

Required the loweſt terms of $\frac{12}{44}$ by finding the greateſt common meaſure.

```
12) 44 (3
    36
    --
     8) 12 (1
         8
         -
      4| 8 (2
         8
         -
```

Hence $\frac{12}{44} \div 4 = \frac{3}{11}$ anſ.

Reduce

Reduce $\frac{112}{116}$ to the lowest terms.

```
112) 116 (1
     112
     ---
       4) 112 (28 
           8
          ---
           32
           32
           --
```

And $\frac{112}{116} \div 4 = \frac{18}{19}$ Anſ.

Reduce $\frac{467}{495}$ to the lowest terms.

```
467) 495 (1
     467
     ---
      28) 467 (16
           28
          ---
          187
          168
          ---
          19) 28 (1
              19
              --
              9) 19 (2
                 18
                 --
               *1) 9 (9
                   9
                   -
```

To

* Here one being the greateſt common meaſure, it appears that the given fraction is in the loweſt terms.

76. To reduce an improper fraction to its whole or mixed number.

RULE.

Divide the numerator by the denominator, and it's done.

EXAMPLES.

Reduce $\frac{76}{52}$ to its whole or mixed number.

$$52)\ 76\ (1\tfrac{24}{52} = 1\tfrac{12}{26} = 1\tfrac{6}{13} \text{ anſwer.}$$
$$\underline{52}$$
$$24$$

Reduce $\frac{76}{4}$ to its whole or mixed number.

Anſwer 19.

Reduce $\frac{76}{5}$ to its whole or mixed number.

Anſwer $15\frac{1}{5}$.

Reduce $\frac{171}{12}$ to its whole or mixed number.

Anſwer $14\ \frac{3}{12} = 14\frac{1}{4}$.

76. To reduce a mixed number to an improper fraction.

RULE.

Multiply the integral part by the denominator of the fractional part, taking in the numerator, the aenominator placed below the product is the anſwer.

EXAMPLES.

Reduce $15\frac{1}{5}$ to an improper fraction.

$15\frac{1}{5}$
$\underline{5}$
76 and $\frac{76}{5}$ is the anſwer.

Reduce

Reduce $72\frac{1}{2}$ to an improper fraction.

Anſwer $\frac{145}{2}$

Reduce $144\ \frac{12}{16}$ to an improper fraction.

Anſwer $144\ \frac{12}{16} = 144\frac{3}{4} = \frac{579}{4}$

Reduce $56\frac{7}{8}$ to an improper fraction.

Anſwer, $\frac{455}{8}$

78. To reduce a whole number to a fraction having a given denominator.

RULE. *Multiply the whole number by the denominator given; placing the ſaid denominator below the product.*

E X A M P L E S.

Reduce 12 to an improper fraction having 4 for the denominator.

$4 \times 12 = 48$ hence ; $\frac{48}{4}$ is the anſwer.

Expreſs 7 as a fraction having 12 for a denominator.

Anſwer $\frac{84}{12}$

Expreſs 25 as a fraction, the denominator being 8.

Anſwer $\frac{200}{8}$

Expreſs 27 as a fraction, the denominator being 9.

Anſwer $\frac{243}{9}$

79. To reduce a compound fraction, to an equivalent ſimple one.

RULE. *Multiply all the numerators continually, and all the denominators continually, the quotients are the new numerator and denominator reſpectively.*

EXAMPLES.

Reduce $\frac{1}{2}$ of $\frac{1}{2}$ to a ſimple fraction.

$$\frac{1 \times 1}{2 \times 2} = \frac{1}{4} \text{ the anſ.}$$

Reduce $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{2}{3}$ to a ſimple fraction.

$$\frac{3 \times 1 \times 2}{4 \times 2 \times 3} = \frac{6}{24} = \frac{3}{12} = \frac{1}{4} \text{ anſwer.}$$

Reduce $\frac{7}{8}$ of $\frac{15}{6}$ of $\frac{5}{12}$ to a ſimple fraction.

Anſwer $\frac{525}{576}$

Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a ſimple fraction

Anſwer, $\frac{1}{5}$.

Reduce $7\frac{1}{2}$ of $\frac{1}{8}$ of $\frac{1}{2}$ to a ſimple fraction.

Anſwer $\frac{15}{32}$

80. To reduce fractions of one integer to another retaining the ſame value.

RULE. *Find how many of the leſs integer make one of the greater, and multiply it with the numerator for a leſs integer, but with the denominator for a greater.*

EXAMPLES.

Reduce $\frac{5}{6}$ of a pound to the fraction of a penny.

$$5 \times 240 = 1200 \text{ hence } \frac{1200}{6} \text{ the anſwer.}$$

Reduce $\frac{7}{12}$ of a ſhilling to the fraction of a farthing.

Anſwer $\frac{336}{12}$

Reduce $\frac{7}{8}$ of a farthing to the fraction of a pound.

Anſwer, $\frac{7}{7680}$

Reduce $\frac{5}{6}$ of a *lb.* to the fraction of a *cwt.*

Anſwer, $\frac{5}{672}$

81. But

81. But if a certain number of the leſs integer do not exactly make one of the greater, then reduce it to ſome integer, which will meaſure the required integer, and from thence to the denomination required.

Reduce $\frac{3}{4}$ of a crown to the fraction of a guinea.

$3 \times 5 = 15$ therefore $\frac{3}{4}$ of a crown is $\frac{15}{4}$ of a ſhilling.

Again, $4 \times 21 = 84$ hence $\frac{15}{84} = \frac{5}{28}$ the anſwer.

Reduce $\frac{7}{10}$ of a ſhilling to the fraction of a quarter guinea.

$7 \times 12 = 84$ that is $\frac{84}{10} = \frac{42}{5}$ of a penny.

Then $5 \times 63 = 315$; hence $\frac{42}{315} = \frac{14}{105}$ of a guinea the anſwer.

82. To reduce fractions of different denominators to equivalent ones, having the ſame denominator.

RULE. *Multiply each numerator into all the denominators but its own for new numerators, and all the denominators continually for a new denominator.*

EXAMPLES.

Reduce $\frac{3}{4}$ and $\frac{1}{2}$ to a common denominator.

Now $\left\{\begin{array}{l} 3 \times 2 = 6 \\ 1 \times 4 = 4 \end{array}\right\}$ Numerators.

And $4 \times 2 = 8$ the denominator; hence $\frac{6}{8}$ and $\frac{4}{8}$ are the anſwer.

Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{8}{9}$ to a common denominator.

$$\left.\begin{array}{l} 3 \times 6 \times 8 \times 9 = 1296 \\ 5 \times 4 \times 8 \times 9 = 1440 \\ 7 \times 4 \times 6 \times 9 = 1512 \\ 1 \times 4 \times 6 \times 8 = 1536 \end{array}\right\} \text{New numerators.}$$

Alſo $4 \times 6 \times 8 \times 9 = 2728$ the denominator; therefore $\frac{1296}{1728}$, $\frac{1440}{1728}$, $\frac{1512}{1728}$, $\frac{1536}{1728}$, the anſwer.*

Reduce $\frac{3}{4}$, $\frac{1}{4}$, $\frac{2}{3}$, and $\frac{4}{5}$ to a common denominator.
Anſwer $\frac{180}{240}$, $\frac{60}{240}$, $\frac{160}{240}$, $\frac{192}{240}$.

Reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{5}{6}$, and $\frac{1}{5}$ to a common denominator.
Anſwer $\frac{120}{360}$, $\frac{90}{360}$, $\frac{300}{360}$, $\frac{72}{360}$.

Reduce $\frac{5}{8}$, $5\frac{1}{2}$ and $\frac{2}{3}$ to a common denominator.
Anſwer $\frac{30}{48}$, $\frac{264}{48}$, $\frac{32}{48}$.

83. To find the value of a vulgar fraction.

RULE. *Multiply the numerator by ſo many of the next inferior denomination as make one in that preceding it, dividing the product by the denominator, continuing the operation to as many denominations as are neceſſary, or till nothing remain.*

EXAMPLES.

Reduce $\frac{3}{4}$ of a pound ſterling to its value.

```
   3
  20
 ----   s.
 4|60  (15 Anſ.
   4
 ----
  20
  20
 ----
   ..
```

What

* The demonſtration of this rule will conſiſt in proving that the fraction retains the ſame value, after being reduced to the common denominator, as before, in order to which let us write the components of the new fractions, inſtead of the fractions themſelves, and we ſhall find that the numerator and denominator are each multiplied alike, for $\frac{3 \times 6 \times 8 \times 9}{4 \times 6 \times 8 \times 9} = \frac{1296}{1728} = \frac{3}{4}$: here the numerator and denominator are each multiplied by 6, 8, 9, and therefore can ſuffer no change in the value, being each increaſed in the ſame proportion, and ſo of the reſt, *Q. E. D.*

What is the value of $\frac{5}{12}$ of a *cwt* ?

$$
\begin{array}{rll}
 & 5 & \\
 & 4 & \\
\hline
 & & qr.\quad lb.\quad oz. \\
12) & 20 & (1 \quad 28 \quad 10\tfrac{2}{3} \text{ anſwer.} \\
 & 12 & \\
\hline
 & 8 & \\
 & 28 & \\
\hline
12) & 224 & (18 \\
 & 12 & \\
\hline
 & 104 & \\
 & 96 & \\
\hline
 & 8 & \\
 & 16 & \\
\hline
12) & 128 & (10 \\
 & 12 & \\
\hline
 & 8 &
\end{array}
$$

Required the value of $\frac{4}{13}$ of a pound ſterling. Anſwer 6 *s.* $2\frac{11}{13}$ *d.*

Required the value of $\frac{2}{15}$ of nine ſhillings. Anſwer 1 *s.* $2\frac{2}{5}$ *d.*

Required the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of a crown. Anſwer 10 *d.*

Required the value of $3\frac{3}{17}$ of a mile. Anſwer 1 *f.* 16 *p.* 2 *yd.* 1 *f.* $9\frac{3}{17}$ *i.*

84. To reduce money weights, &c. to fractions.

RULE. *Reduce the given quantities to the loweſt denomination mentioned, placing as many of the ſaid denomination beneath the reduced quantity as make* 1 *of the required integer.*

EXAMPLES.

Reduce 7 *s.* $6\frac{1}{2}$ *d.* to the fraction of a pound sterling.

$$\begin{array}{l} 12 \\ \hline 90 \\ 2 \\ \hline 181 \end{array}$$

hence per rule $\frac{181}{480}$ is the answer.

Reduce $10\frac{1}{2}$ *d.* to the fraction of a pound sterling.

$$\begin{array}{l} 2 \\ \hline 21 \end{array}$$

hence $\frac{21}{480} = \frac{7}{160}$ the answer.

Reduce 2 *qr.* 14 *lb.* to fraction of a *cwt.*

Answer $\frac{70}{112} = \frac{5}{8}$.

What fraction of a *cwt.* is equivalent to 1 *qr.* 21 *lb.* 8 *oz*?

Answer $\frac{99}{224}$.

Reduce 3 *oz.* 14 *dwt.* 20 *grs.* to the fraction of a pound troy.

Answer $\frac{1220}{5760} = \frac{122}{576} = \frac{61}{288}$.

Reduce 7 *s.* 6 *d.* to the fraction of a mark.

Answer $\frac{90}{160} = \frac{9}{16}$.

ADDITION *and* SUBTRACTION *of* VULGAR FRACTIONS.

RULE.

REDUCE compound fractions to simple ones, mixed numbers to improper fractions, different integers to the same, and all to a common denominator; the sum or difference of the numerators placed over the common denominator, is the answer.

EXAMPLES.

Add $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ together.

$$\begin{array}{r} 1 \times 4 \times 5 \times 6 = 120 \\ 1 \times 3 \times 5 \times 6 = 90 \\ 1 \times 3 \times 4 \times 6 = 72 \\ 1 \times 3 \times 4 \times 5 = 60 \\ \hline \text{Sum } 342 \end{array}$$

$3 \times 4 \times 5 \times 6 = 360$ the common denominator: hence $\frac{342}{360} = \frac{57}{30} = \frac{19}{20}$ the anſwer.

To $\frac{3}{5}$ of a crown add $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of a pound.

$\frac{3}{5}$ of $\frac{1}{4} = \frac{3}{20}$ and $\frac{3 \times 4 \times 5}{4 \times 5 \times 6} = \frac{3}{6} = \frac{1}{2}$; then $\left\{\begin{array}{l} 3 \times 2 = 6 \\ 1 \times 5 = 5 \end{array}\right.$

11 the numerator, and 2×20 the common denominator: hence $\frac{11}{40} = 5\,s.\ 6\,d.$ the anſwer.

Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$ and $7\frac{1}{2}$ together.

Anſwer $\frac{764}{80} = 9\frac{22}{40} = 9\frac{11}{20}$

Suppoſe I buy $\frac{3}{8}$ of a ſhip, and afterwards $\frac{5}{16}$ more, what ſhare have I then? Anſwer $\frac{11}{16}$.

Required the ſum of $\frac{5}{4}$ of $\frac{2}{3}$ and $\frac{3}{6}$ of 12?

Anſwer $6\frac{5}{6}$.

Take $\frac{3}{5}$ from $\frac{5}{6}$.

$$\begin{array}{r} 5 \times 5 = 25 \\ 3 \times 6 = 18 \\ \hline \text{Dif. } 7 \end{array}$$

$5 \times 6 = 30$ the common denominator; hence $\frac{7}{30}$ the anſwer.

Take

Take $\frac{3}{4}$ of $\frac{2}{3}$ from $5\frac{1}{8}$.

$\left.\begin{array}{r}\frac{3}{4} \text{ of } \frac{2}{3} = \frac{3}{6} = \frac{1}{2} \\ \text{and } 5\frac{1}{8} = \frac{41}{8}\end{array}\right\}$ now $\left.\begin{array}{r}41 \times 2 = 82 \\ \text{and } 1 \times 8 = 8\end{array}\right\}$ the numerators;

and $8 \times 2 = 16$ the common denominators: hence

$\frac{82}{16} - \frac{8}{16} = \frac{82 - 8}{16} = \frac{74}{16} = \frac{37}{8} = 4\frac{5}{8}$ the anſ.

MULTIPLICATION *of* VULGAR FRACTIONS.

RULE.

86. *REDUCE mixt numbers to improper fractions, and fractions of different integers to the ſame; then multiply all the numerators together, and all the denominators together, the products are the numerator and denominator of the required fraction.*

EXAMPLES.

Multiply $\frac{7}{8}$, $\frac{3}{4}$ and $\frac{5}{6}$ together.

Here $\frac{7 \times 3 \times 5}{8 \times 4 \times 6} = \frac{105}{192} = \frac{35}{64}$ the anſwer.

Multiply $7\frac{1}{2}$ by $6\frac{1}{2}$.

Now $7\frac{1}{2} = \frac{15}{2}$ and $6\frac{1}{2} = \frac{13}{2}$

But $\frac{15 \times 13}{2 \times 2} = \frac{195}{4} = 48\frac{3}{4}$ the anſwer.

Multiply 2 *s.* 6 *d.* by 2 *s.* 6 *d.* a ſhilling the integral quantity.

2 *s.* 6 *d.* $= \frac{30}{12} = \frac{5}{2}$ of a ſhilling; therefore $\frac{5 \times 5}{2 \times 2} =$

$\frac{25}{4} = 6\frac{1}{4} =$ 6 *s.* 3 *d.* Anſwer.

Multiply

Multiply 2*s*. 6*d*. by 2*s*. 6*d*. a pound the integeral quantity.

$2s. 6d. = \frac{30}{240} = \frac{3}{24} = \frac{1}{8}$ of a *l*. and $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64} =$ $3\frac{3}{4}d$. Anf. *

How much is 20 made lefs when multiplied by $\frac{3}{4}$ of $\frac{1}{2}$? Anfwer $12\frac{1}{2}$.

Required the product of 3 *l*. 10 *s*. 6*d*. by 5*d*. Anfwer 1 *s*. $5\frac{5}{8}d$.

Multiply 2*l*. 10*s*. 6*d*. by 3*l*. 12*s*. $9\frac{1}{2}d$. Anfwer 9*l*. 3*s*. 9*d*. $2\frac{7}{20}f$.

DIVISION *of* VULGAR FRACTIONS.

RULE.

87. *PREPARE the fractions as in multiplication, (article 86) then invert the divifor † and perform the work by multiplication of vulgar fractions.*

EXAMPLSS.

Divide $37\frac{1}{2}$ by $42\frac{3}{4}$.

$37\frac{1}{2} = \frac{75}{2}$ and $42\frac{3}{4} = \frac{171}{4}$ now per rule $\frac{4 \times 75}{171 \times 2} =$ $\frac{300}{342} = \frac{150}{171} = \frac{50}{57}$ the anfwer.

Divide 15*s*. 6*d*. by 3*s*. 4*d*. a pound the integer.

$15s.\ 6d. = \frac{31}{40}$ of a £. and $3s.\ 4d. = \frac{1}{6}$ of a £.

and $\frac{31 \times 6}{40 \times 1} = \frac{186}{40} = 4\frac{26}{40} = 4l.\ 13s.$ the anf.

What

* Let us here take notice, that when either factor is lefs than the integer, the product is proportionally leffened by multiplication; for $20 \times 1 = 20$, but if 20 is multiplied by lefs than an unit, the quotient will be lefs than 20.

† By inverting the divifor, you alfo invert the value thereof; and therefore it is, that the work muft be performed by multiplication.

What number multiplied by 20 will give $12\frac{1}{2}$?

Anſwer $\frac{5}{8}$.

Divide 1 *s.* $5\frac{5}{8}$ *d.* by 5 *d.* a pound the integer.

Anſwer 3 *l.* 10 *s,* 6 *d.*

Divide 2 *s.* 6 *d.* by $2\frac{1}{4}$ *d.* a pound the integer.

Anſwer 13 *l.* 6 *s.* 8 *d.*

The RULE *of* THREE *in* VULGAR FRACTIONS.

88. WE ſhall not in this place take any notice of the difference between direct and inverſe proportion, but ſuppoſing the learner already acquainted with their properties in whole numbers proceed to examples.

If $\frac{3}{8}$ of velvet coſt 8 *s.* what will $\frac{5}{16}$ coſt?

$$\text{Now it will be as } \overset{yd.}{\frac{3}{8}} : \overset{l.}{\frac{8}{20}} :: \overset{yd.}{\frac{5}{16}} : \overset{l.}{\frac{8 \times 8 \times 5}{3 \times 20 \times 16}} =$$

$$\frac{8 \times 8}{3 \times 16} = \frac{8}{6} = 1\,l.\ 6\,s.\ 8\,d. \text{ the anſwer.}$$

If $3\frac{1}{8}$ ounces of ſilver coſt 6 *s.* 8 *d.* per ounce, what does it come to?

$$\overset{oz.}{1} : \overset{l.}{\frac{1}{3}} :: \overset{oz.}{\frac{25}{8}} : \overset{l.}{\frac{25}{24}} = 1\,l.\ 0\,s.\ 10\,d. \text{ anſ.}$$

Required the purchaſe of 1230 *l.* bank ſtock at $108\frac{5}{8}$ *l.* per cent?

$$\text{Now as } 100\,l. : 108\tfrac{5}{8}\,l. \left(= \frac{869}{8}\right) :: 1230\,l. :$$

$$\frac{869 \times 1230\,l.}{8 \times 100} = \frac{869 \times 123}{80} = \frac{106887}{80} = 1336\,l.$$

1 *s.* 9 *d.* Anſwer.

If 5 men do a piece of work in $7\frac{1}{2}$ days, in what time will 8 men do the ſame?

$$\overset{m.}{5} : \overset{d.}{\frac{15}{2}} :: \overset{m.}{8} \cdot \overset{d.}{\frac{5 \times 15}{8 \times 2}} = \frac{75}{16} = 4\,\frac{11}{16} \text{ the anſwer.}$$

Suppoſe

Suppofe 1 *lb.* of indigo coft 7 *s.* 3 *d.* what will 848 *lb.* coft ?

Here 1 *lb.* $\frac{29\,l.}{80}$ :: 848 *lb.* : $\frac{29 \times 848}{80} = \frac{29 \times 106}{10} =$

$\frac{29 \times 53}{5} =$ 307 *l.* 8 *s.* the anfwer.

Thefe examples are fufficient to fhew the utility of vulgar fractions in contracting the operations in common arithmetic. Our plan leads us now to decimal arithmetic, to which we proceed.

DECIMAL FRACTIONS.

DIFINITION.

89. A Decimal is a fraction, the denominator of which is an unit with as many cyphers annexed to it as there are digits in the numerator, as $\frac{2}{10}$, $\frac{75}{100}$, &c. but they are always written without their denominators. The genefis of decimals will be beft conceived by deducting them from vulgar fractions.

REDUCTION *of* DECIMALS.

90. TO reduce a vulgar fraction to an equivalent decimal.

RULE. *Divide the numerator with cyphers annexed thereto by the denominator, dafhing as many figures off in the quotient to the right hand, (by a comma) as there are cyphers annexed to the numerator or dividend.*

EXAMPLES.

Reduce $\frac{1}{4}$ to a decimal.

4) 1·00

·25 anf.

Reduce

Reduce $\frac{1}{2}$ to a decimal.

$$\begin{array}{r} 2)\ 1{\cdot}0 \\ \hline {\cdot}5 \text{ Anſ.} \end{array}$$

Reduce $\frac{3}{4}$ to a decimal.

$$\begin{array}{r} 4)\ 3{\cdot}00 \\ \hline {\cdot}75 \text{ Anſ.*} \\ \hline \end{array}$$

Reduce $\frac{25}{32}$ to a decimal.

$$\begin{array}{r} 32)\ 25{\cdot}000000\ ({\cdot}784375 \text{ Anſ.} \\ 224 \\ \hline 260 \\ 256 \\ \hline 140 \\ 128 \\ \hline 120 \\ 96 \\ \hline 240 \\ 224 \\ \hline 160 \\ 160 \\ \hline \end{array}$$

Reduce the fraction $\frac{1}{3}$ to a decimal.

$3)\ \frac{,00000}{33333}$: It is eaſy to conceive that if this operation was continued to infinity, the 3 would always repeat in the quotient; hence ſuch are ſeldom continued further than the firſt repeating digit, putting a point above it, to ſignify its repeating, and therefore the quotient or anſwer will ſtand thus. $\dot{3}$

Reduce

* From theſe three examples we find that ·25 is the decimal of a quarter ·5 of a half and ·75 of three quarters.

Reduce $\frac{257}{999}$ to a decimal.

$$
\begin{array}{r|l}
999) & 257{\cdot}000 \; (\dot{2}5\dot{7} \\
 & 1998 \\
\hline
 & 5720 \\
 & 4995 \\
\hline
 & 7250 \\
 & 6993 \\
\hline
 & 257 \\
\hline
\end{array}
$$

The figures 257 in this example would always recur and are therefore marked as in the quotient, and termed repeatends, or compound circulates.

Reduce $\frac{26}{32}$ or $\frac{13}{16}$ to a decimal. Anſwer ·8125.

Reduce $\frac{1}{5}$ to a decimal. Anſwer ·2.

Reduce $\frac{1}{42}$ to a decimal. Anſwer $\cdot 0\dot{3}809\dot{5}$.

Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to a decimal.

Anſ. $\cdot 1\dot{6}$.

91. TO reduce compound quantities, as money, &c. to equivalent decimals.

RULE 1. *Reduce the given quantity to a vulgar fraction, (by art. 84) and then to a decimal (per laſt art.)*

RULE 2. *Write the different denominations under each other, the loweſt denomination uppermoſt, &c. for dividends, and againſt each of theſe write ſo many as make one of the next ſuperior denomination for diviſors, and let a line be drawn between them; then begin to divide the uppermoſt annexing or ſuppoſing cyphers, annexed to the dividend, and place the quotient as decimals to the denomination below; the laſt quotient ſo ordered is the anſwer.*

EXAMPLES.

Reduce 12*s.* 6*d.* to a decimal.

$12s.\ 6d. = \frac{150}{240} = \frac{15}{24} = \frac{5}{8}l.$

8) 5·000 (·625 Ans.
48
——
20
16
——
40
40
——

Or per Rule 2d
12 | 6·
20 | 12·5
·625 Ans.

Required the decimal of 10*s.* $6\frac{1}{2}$*d.*

$10s.\ 6\frac{1}{2}d. = \frac{253}{480}$

480) 253·00000 (·52708$\dot{3}$ Ans.
2400
——
1300
960
——
3400
3360
——
4000
3840
——
1600
1440
——
160

Per 2d Rule
2 | 1
12 | 6·5
2|0 | 10·541$\dot{6}$*
52708$\dot{3}$ Ans.

Reduce 8*s*, $9\frac{1}{2}$*d.* to the decimal of a pound sterling.

2 | 1
12 | 9·5
20 | 8·791$\dot{6}$
·43958$\dot{3}$ Answer.

Reduce

* This being a circulating digit in dividing by 20, we suppose 6s. instead of cyphers annexed, which must be carefully observed in similar cases.

Reduce $6\frac{1}{2}d.$ to the decimal of a pound sterling.

2	1·
12	6·5
20	·541$\dot{6}$
	·02708$\dot{3}$* Answer.

Reduce 3*qr.* 14*lb.* to the decimal of a *cwt.*

7	14·
4	(2·)
4	3·5
	·875 Answer.

Required the decimal of 12*s.* $8\frac{1}{2}d$? Anſ. ·63541$\dot{6}$.

Reduce 5*oz.* 6*dwt.* 17*grs.* to the decimal of a pound troy. Anſwer ·444618$\dot{0}$5.

Reduce 5*m.* 3*w.* 5*d.* to the decimal of a year. Anſwer ·456043 +.

92. To find the value of a decimal fraction.

RULE. *Multiply the given decimal by ſo many of the next inferior denomination, as make* 1 *in that preceding it, pointing off as many figures in the product for decimals as there are decimals in the multiplicand; ſo proceeding, thro' every denomination till the work is ſufficiently exact, or till nothing remain, the figures on the left ſide of the point, being the different denominations required.*

* It will not be difficult to comprehend from the method of reducing quantities to decimals, that the notation is contrary to that of whole numbers, the firſt figure after the point being of moſt value, and every remove form it in a tenfold ratio inferior to that preceding; whereas whole numbers increaſe in the ſame proportion, according to the diſtance from the ſaid point; and hence we obſerve, that the cypher prefixed to the figures in the example decreaſes the value to one tenth of what it was before the cypher was written. The firſt place after the point is tenths, the ſecond hundreds, the third thouſands, &c.

EXAMPLES.

Required the value of ·634375 of a pound sterling ?

$$\begin{array}{r} 20 \\ \hline 12{\cdot}687500^{*} \\ 12 \\ \hline 8{\cdot}2500 \\ 4 \\ \hline 1{\cdot}00 \end{array}$$

Answer 12*s.* 8¼*d.*

Reduce $\cdot\dot{3}$ of a shilling to its value.

$$\begin{array}{r} \dot{3} \\ 12 \\ \hline 4{\cdot}0\dagger \end{array}$$

Answer 4*d.*

Required the value of $\cdot 6\dot{5}$ ‡ of a pound troy

$$\begin{array}{r} 12 \\ \hline 7{\cdot}8\dot{6} \\ 20 \\ \hline 17{\cdot}\dot{3} \\ 24 \\ \hline 1\dot{3} \\ \dot{6}6 \\ \hline 8{\cdot} \end{array}$$

	oz.	*dwt.*	*grs.*
Ans.	7	17	8

Required

*The cyphers at the right hand of the decimals being of no value are omitted, as they occur in the operation.

† If we consider that the vulgar fraction equivalent to $\cdot\dot{3}$ is $\frac{3}{9}$ and that the denominator of circulating digits are 9*s.* we will easily admit that to multiply them into a finite quantity, there must 1 be carried at 9 for $\frac{3}{9} \times 12 = \frac{36}{9} = 4$ agreeable to the example.

‡ If $\cdot 6\dot{5}$ be taken $= \cdot 6\frac{5}{9}$ we shall have $\cdot 6\frac{5}{9} \times 12 = 7{\cdot}2 + \frac{60}{9} = 7{\cdot}8\frac{6}{9}$ and $\cdot 8\frac{6}{9} \times 20 = 16{\cdot}0 + \frac{120}{9} = 17{\cdot}\frac{3}{9}$ again, $\frac{3}{9} \times 24 = \frac{72}{9} = 8{\cdot}$ which is also agreeable to the example: we may here take notice, that in multiplying by 24 when we come to add 'tis necessary to make the lines end together, which does not alter the value of the circulates.

Required the value of ·75 of a *cwt.* Anſwer 3 *qr.* 0 *lb.*

What is the value of ·7525 of a pound ſterling.* Anſwer 15 *s.* 0 *d.* 2·4 *f.*

Required the value of ·365 of a mile. Anſwer 2 *f.* 36 *p.* 4 *y.* 1·2 *f.*

ADDITION and SUBTRACTION of DECIMALS.

93. RULE. *Write the propoſed numbers to be added under each other, in ſuch ſort that units ſtand under units, tens under tens, tenths under tenths, &c. taking the ſum or difference (by art.* 12 *and* 16*) and place the decimal point in a perpendicular ſituation, with reſpect to the given quantities.*

EXAMPLES.

Add 74·647 + 794·67 + 7·23 + ·4846 + 94·6765 + 497655·8 + 46874 together. These rightly placed ſtand thus:

$$\begin{array}{r@{}l}
74 & \cdot647 \\
794 & \cdot67 \\
723 & \cdot \\
 & \cdot4846 \\
94 & \cdot6765 \\
497655 & \cdot8 \\
46874 & \\
\hline
546217 & \cdot2781 \text{ Sum} \\
\hline
\end{array}$$

I 3 What

* The value of a pound ſterling may be found by inſpection, thus double the place of tenths for ſhillings, and if the hundredths place exceed 5, add 1 to the ſhillings, calling the exceſs of the hundreds place with the thouſandths ſo many farthings, abating one in twenty-four.

EXAMPLE. Required the value of ·275: here 2 doubled is 4 ſhillings, and becauſe in the hundredths there is a 7, we add 1 to the ſhillings which makes 5, and what remains in the ſecond and third place, viz. 25 call farthings, rejecting 1 in 24 there will be 24 farthings = to 6 *d.* therefore the anſwer is 5 *s.* 6 *d.*

What is the sum of $7694{\cdot}\dot{3}$ + $7{\cdot}652$ + $76{\cdot}947$ + $894{\cdot}75\dot{6}$ + $4{\cdot}769276$ + $56{\cdot}34\dot{6}$.

$$
\begin{array}{r}
\dot{7}694{\cdot}3333333 \\
7{\cdot}6520000 \\
76{\cdot}94\dot{7}0000 \\
894{\cdot}75\dot{6}6666 \\
4{\cdot}769276\dot{0}^{*} \\
56{\cdot}34\dot{6}6666 \\
\hline
8734{\cdot}804942\dot{6} \text{ Answer.}
\end{array}
$$

$$
\begin{array}{rl}
\text{From} & 46{\cdot}52 \text{ take } 42{\cdot}756 \\
 & 42{\cdot}756 \\
\hline
\text{Difference} & 3{\cdot}764
\end{array}
$$

$$
\begin{array}{rl}
\text{From} & 856{\cdot}234 \text{ take } 37{\cdot}2785 \\
 & 37{\cdot}2785 \\
\hline
 & 818{\cdot}9555 \text{ Answer.}
\end{array}
$$

From

* When circulates and finite quantities are added or ſubtracted, the finite quantities may be made to circulate by annexing cyphers to them, and the circulates muſt be continued till they end in a perpendicular column with the finite quantities: this done, proceed to take the ſum or difference, carrying at 9 the firſt column. If we conſider that by this device there will be nothing but circulates in the firſt column, and that their denominator is 9, the method will be accounted for.

From $176{\cdot}54\dot{3}$ take $76{\cdot}54687$

$$
\begin{array}{r}
176{\cdot}54\dot{3}333 \\
79{\cdot}54687\dot{0} \\
\hline
99{\cdot}99646\dot{3} \text{ Anf.}
\end{array}
$$

From $72{\cdot}96\dot{5}$ take $56{\cdot}\dot{8}$

$$
\begin{array}{r}
56{\cdot}\dot{8}88 \\
\hline
16{\cdot}07\dot{6} \text{ Anfwer.}
\end{array}
$$

From $76{\cdot}9$ take $74{\cdot}956\dot{8}$

$$
\begin{array}{r}
76{\cdot}9\dot{0}0 \\
74{\cdot}96\dot{8} \\
\hline
1{\cdot}93\dot{1} \text{ Anf.} \\
\hline
76{\cdot}90\dot{0} \text{ Proof.}
\end{array}
$$

MULTIPLICATION *of* DECIMALS.

94 RULE I. *Multiply the factors as in whole numbers, pointing off as many decimals in the product as there are in both factors; but if the product contain not so many, the defect must be supplied with cyphers prefixed to the said product.*

EXAM-

EXAMPLES.

Multiply	74·653	Multiply	·14687
By	5·61	By	·25
	74653		73435
	447918		29374
	373265		·0367175† Anſwer.
Product	418·80333*		

95 RULE 2. *When the right hand digit of the multiplicand is a circulate, carry at 9 the firſt figure of every line, pointing the digit, ſet down for a circulate and take the ſum as directed (per note to art. 93) in addition.*

EXAMPLES.

Multiply	$6·46\dot{5}$	Multiply	$67·6\dot{3}$
By	17·69	By	2·547
	$5819\dot{0}$		$4734\dot{3}$
	$3879\dot{3}3$		$2705\dot{3}3$
	$452\dot{5}888$		$338\dot{1}666$
	$646\dot{5}555$		$1352\dot{6}666$
	$114·3756\dot{7}$ Anſwer.		$172·2621\dot{0}$‡

RULE

*DEMONSTRATION.

Let the given factors be taken in their equivalent mixed numbers, viz. $74\frac{653}{1000}$ and $5\frac{61}{100}$ equal to $\frac{74653}{1000}$ and $\frac{561}{100}$ being multiplied, the product is $\frac{41880333}{100000} = 418\frac{80333}{100000}$ and rejecting the denominator, we have 418·80333 agreeable to the above rule.

† What was ſaid concerning multiplication of vulgar fractions, holds true in decimals, viz. that if either factor is leſs than unity, the product is in ſuch proportion leſs than the other factor.

‡ From this example it appears that if a finite quantity is multiplied into an infinite one, the product may be finite.

96 RULE 3. *If the multiplier have a circulate, multiply with it as if a finite quantity; but removing the Product a place more to the left, and then divide it by 9.*

EXAMPLES.

Multiply	$74{\cdot}67\dot{8}$	Multiply	$47{\cdot}6\dot{3}$
By	$\dot{5}$	By	$28{\cdot}4\dot{5}$
	9|373390		9|23815
Product	$4148\dot{7}7$		$2646\dot{1}$
			19052
			38104
			9526
			$1355{\cdot}338\dot{1}$ Ans.

97 RULE 4. *If in each factor there be a circulate, they must be managed as directed by the two last rules.*

EXAMPLES.

Multiply	$7{\cdot}4\dot{6}$
By	${\cdot}5\dot{4}$
	9|$298\dot{6}$
	$33\dot{1}8\dot{5}185$
	$3733\dot{3}3333$
	$4{\cdot}06\dot{5}1\dot{8}518$ Ans.

Multiply

$$\begin{array}{lr}
\text{Multiply} & 76\cdot56\dot{8} \\
\text{By} & \cdot4\dot{5} \\
\hline
 & 9|38284\dot{4} \\
\hline
 & 425\dot{3}82916049\dot{3}8 \\
 & 30627\dot{5}55555555 \\
\hline
 & 34\cdot881\dot{3}8471604\dot{9}3 \text{ Anſwer.} \\
\hline
\end{array}$$

What number divided by ·25 will give 800? Anſwer 200.

What number divided by ·48 will increaſe to 420? Anſwer 201 6.

Required the product of $75\cdot6\dot{8}$ by $\cdot\dot{3}$? Anſwer $25\cdot\dot{2}29629\dot{6}$.

DIVISION *of* DECIMALS.

98 RULE I. *The operation is performed as in whole numbers, and there muſt be as many decimal places made in the quotient, as added to thoſe in the diviſor, will equal theſe in the dividend; or which is the ſame thing, count how many decimal places the dividend exceeds the diviſor, and ſo many muſt be pointed of in the quotient for decimals. If there is not a ſufficient numbers of figures in the quotient to anſwer the purpoſe, cyphers muſt be prefixed on the left ſide till there is: but if the decimal places in the diviſor exceed thoſe in the dividend, cyphers muſt be annexed to the dividend till at leaſt they are equal, and if neceſſary more.*

Ex-

EXAMPLES.

```
7·5) 47·625 (6·35
     450
     ---
      262
      225
      ---
       375
       375
       ---
```

$17)\ 62\cdot\dot{3}^{*}\ (3\cdot\dot{6}$

```
17) 62·3* (3·6
    51
    ---
    113
    102
    ---
     11
```

```
4·6) ·36754 (·0799
      322
      ---
       455
       414
       ---
        414
        414
        ---
```

```
275) 8·46500 (·03078, &c.
     825
     ----
      2150
      1925
      ----
       2250
       2200
       ----
         50
         --
```

Divide

* In cases of this nature, where we find the quotient will not soon become a circulate, we continue to carry forward the operation by bringing down cyphers or the circulating digit, till the work is sufficiently exact.

Divide 418·80333 by 74·653. Anſwer 5·61.

Required the quotient when the dividend is ·0367175 and diviſor ·25. Anſwer ·14687.

What number multiplied by 756 will make 320 Anſwer ·42328, &c.

RULE 2. *If the diviſor be a circulate, write down the diviſor and dividend in order of diviſion, and under theſe write them a ſecond time, each removed to the right hand, ſo many places as there are circulating digits in the diviſor; in this ſituation take the under lines from the upper ones, making the differences a new diviſor and dividend, and the quotient will be the anſwer.*

EXAMPLES.

$$
\begin{array}{rl}
28\cdot4\dot{5}) & 1355\cdot338\dot{1} \\
2\cdot84) & 135\cdot5338 \\
\hline
25\cdot61) & 1219\cdot8043 \;(47\cdot63 \text{ Anſ.*} \\
 & 10244 \\
\hline
 & \;\;19540 \\
 & \;\;17927 \\
\hline
 & \;\;\;16134 \\
 & \;\;\;15366 \\
\hline
 & \;\;\;\;\;7683 \\
 & \;\;\;\;\;7683 \\
\hline
\end{array}
$$

Divide 114·375$\dot{6}$7 by 6·46$\dot{5}$. Anſwer 17·69.

Divide 172·2621 by 67·6$\dot{3}$. Anſwer 2·547.

The

* By comparing this example with the ſecond example, rule 3 in multiplication, it will be found that they prove each other, and ſo may the truth of any of the other examples be examined.

The more complicated cases in Multiplication and Division we pass over, judging them easier performed by vulgar fractions, from whence the several rules we have delivered concerning repeating or infinite decimals are deduced, by means of this single consideration, viz. that the denominators of circulating decimals consist of so many nines as there are recurring digits in the decimal fraction. I shall not in this place (as is common) treat of the square and cube roots, &c. but defer them until the reader can be assisted with a proper rationale of them, from whence he may deduce the rules; therefore I now proceed to the remaining useful rules in arithmetic, as proper exercises to the foregoing principals.

BARTER *or* TRUC.

Definition.

100. BARTER is the exchanging one or more kinds of goods or commodities for those of some others.

As this rule is only a further illustration of multiplication of compound quantities, the rule of three direct and practice, we shall not puzzle the learner, or swell the book with particular rules, but rather leave him to an inspection of the examples and his own judgment, advising him to use the shortest methods, and endeavour to make the subject of arithmetic (if I may be indulged the phrase) consonant to common sense.

Examples.

How much rum at 8*s*. 6*d*. per gallon may be had for 27 yards of broad cloth at 17*s*. 6*d*. per yard?

27*yds*. at		17*s*.	6*d*.
			9
	7	17	6
			3
£.	23	12	6

Then 8*s*. 6*d*. : 1 *gal*. :: 23 *l*. 12*s*. 6*d*. : $\frac{23\,l.\ 12\,s.\ 6\,d.}{8\,s.\ 6\,d}$ that is $\frac{23{\cdot}625}{{\cdot}425} = \frac{4{\cdot}725}{{\cdot}085} = \frac{{\cdot}945}{{\cdot}017} =$ 55·588 *galls*, &c. the answer.

What quantity of indigo at 6*s*. 8*d*. per *lb*. is equal in value to 112 *lb*. of tea at 7*s*. 6*d*. per *lb*?

$7s.6d. = \frac{3}{8}$ and 1 *lb*. : $\frac{3}{8}l.$ :: 112 *lb*. : $\frac{112 \times 3}{8} = 14 \times 3 = 42 l.$

$6s.8d. = \frac{1}{3}$ also $\frac{1}{3} l.$: 1 *lb*. :: 42 *l*. : $42 \times 3 = 126$ *lb*. the anſ.

Required what quantity of canvas at $7\frac{1}{2}d$. per yard, may be given for 5 *cwt*. of cheeſe at 1 *l*. 9 *s*. 6 *d*. per *cwt*?

$$1 l.\ 9 s.\ 6 d. = \frac{59}{40}$$

$$7\tfrac{1}{2} = \frac{15}{480} = \frac{3}{96} = \frac{1}{32}$$

and $\frac{1}{32} l.$: 1 *yd*. :: $\frac{59 \times 5}{40} l. : \frac{59 \times 5 \times 33}{40} yd.$

$$= \frac{59 \times 32}{8} = 59 \times 4$$

$= 236$ *yds*. the anſ.

C and *D* barter, *C* has 53 quarters of corn at 1 *l*. 10 *s*. per quarter, for which *D* would give 13 *cwt*. 0 *qr*. 16 *lb*. of ſugar at 4 *l*. 12 *s*. per *cwt*. and the balance in raiſins at $6\frac{1}{2}d$. per *lb*. how many *lb*. of raiſins muſt be given?

Now

$$1 l.\ 10 s.\ 0 d. = \frac{3}{2} l.$$

$$13 cwt.\ 0 qr.\ 16 lb. = \frac{1472}{112} = \frac{92}{7} cwt.$$

$$4 l.\ 12 s.\ 0 d. = \frac{92}{20} = \frac{23}{5} l.$$

$$6\tfrac{1}{2} d. = \text{- -} \frac{13}{480} l.$$

then 1 *qr*. : $\frac{3}{2}l.$ :: 53 *qr*. : $\frac{53 \times 3}{2} = \frac{159}{2} = 79 l.\ 10 s.$

and 1 *cwt*. : $\frac{23}{5} l.$ :: $\frac{92}{7}$ *cwt*. : $\frac{23 \times 92}{5 \times 7} l. = \frac{2116}{35} = 60\ \ 9\frac{1}{7}$

Difference $19\ \ 0\frac{6}{7}$

Now $19 l.\ 0\frac{6}{7}s. = 19\frac{3}{70} = \frac{1333}{70} l.$

And

and $\frac{13}{480}l.$: $1\,lb.$ :: $\frac{1333}{70}l.$: $\frac{480 \times 1333}{70 \times 13}$ = $\frac{48 \times 1333}{7 \times 13}$ = $\frac{63984}{91}$ = $703\frac{11}{91}\,lb.$ the anſwer.

If $5\frac{1}{4}$ *cwts*. of tobacco, at 1 *l*. 18 *s*. per *cwt*. be given for 24 yards of cloth, what is the value of the cloth per yard. Anſwer 8 *s*. $3\frac{3}{4}$ *d*.

LOSS *and* GAIN.

DEFINITION.

101. QUESTIONS are propoſed in this rule, which determine the gain or loſs of commodities, and being nothing but the application of the golden rule, to caſes of this nature we proceed to

EXAMPLES.

Bought 32 *cwt*. 2 *qr*. of flax at 2 *l*. 2 *s*. 6 *d*. per *cwt*. and ſold it for 2 *l*. 10 *s*. per cwt. what was gained thereby?

	l.	*s*.	*d*.
From	2	10	0
Take	2	2	6
Dif.		7	6 = $\frac{3}{8}l.$

cwt. *l*. *cwt*.

and 1 : $\frac{3}{8}$ :: $\frac{65}{2}$: $\frac{3 \times 65}{2 \times 8}$ = 12 *l*. 3 *s*. 9 *d*. the anſwer.

Bought a quantity of ſugar at 5 *l*. 15 *s*. per *cwt*. how muſt it be ſold per *lb*. to gain 18 *l*. per cent?

5 *l*. 15 *s*. = 5·75 *l*. then 100 *l*. : 118 *l*. :: 5·75 *l*. : $\frac{118 \times 5{\cdot}75}{100}l.$ = 118 × ·0575 and 112 *lb*. : 118 × ·0575 :: 1 *lb*. : $\frac{118 \times {\cdot}0575}{112}$ = $\frac{59 \times {\cdot}0575}{56}$ = $\frac{59 \times 0115}{11{\cdot}2}$ = $\frac{59 \times {\cdot}0023}{2{\cdot}24}$ = $\frac{{\cdot}1357}{2{\cdot}24}$ = ·06058, &c. = 1 *s*. $2\frac{1}{2}$ *d*. per *lb*.

Sold goods at 5 *s.* 6 *d.* and thereby gained 26 *l.* per cent. what will I gain per cent. when the same quantity is sold at 5 *s*?

$5s.\ 6d. = 5{\cdot}5s.$ and $5{\cdot}5s. : 126l. :: 5s. : \frac{126 \times 5}{5{\cdot}5}$
$= \frac{126}{1{\cdot}1} = 114{\cdot}\dot{5}\dot{4} = 114l.\ 10s.\ 10\frac{10}{11}d.$ Hence 14 *l.* 10 *s.* $10\frac{10}{11}$ *d.** is the required gain per cent.

EXCHANGE.

DEFINITION.

102. EXCHANGE is the receiving of money in one country for the same value paid in another, and is above or below par † according to the circumstances of trade and places.

103. In Ireland, America, and the West Indies, the accompts are kept in pounds, shillings, &c. the par with Ireland being $108\frac{1}{3}$ *l.* Irish per cent. sterling, and 7 *l.* in the West Indies is valued at 5 *l.* sterling.

104. In Holland, Flanders, and Germany, they keep their accompts sometimes in pounds, shillings, &c. the par being 33 *s.* 4 *d.* flemish per pound sterling, and sometimes in guilders, stivers, and pennings.

Note, that	8	pennings	make	1 groat or penny.
	2	groats	——	1 stiver.
	6	stivers	——	1 schilling.
	20	stivers	——	1 guilder.
	20	schillings or	}	1 pound flemish.
	6	guilders	}	

In

* Questions of this nature are wrong solved in several books of arithmetic.

† Par is the intrinsic value of a quantity of money in one place, compared with an equivalent in another.

105. In France accompts are kept in livers, ſols, and Deniers.

12 deniers	making	1 ſol
20 ſols	——	1 livre
3 livres	——	1 crown, equal to

4*s.* 6*d.* at par.

106. In Spain accompts are kept in piaſters, rials, and marvadies. Note 372 marvadies make 1 rial.

8 rials —— 1 piaſter.

the par is at 4*s.* 6*d.* per piaſter.

107. In Portugal 1000 reas make 1 milrea, by which they exchange, the par being about 6*s.* 9*d.*

EXAMPLES.

From London is remitted to Dublin 567*l.* 10*s.* 6*d.* What muſt be received exchange at 110*l.* 16*s.* per cent?

$$
\begin{array}{r}
100 : 110{\cdot}8 :: 567{\cdot}525 : \\
110{\cdot}8 \\
\hline
4540200 \\
6242775 \\
\hline
628{\cdot}817700 = 628 \quad 16 \quad 4{\cdot}248
\end{array}
$$

(*l.* *s.* *d.* above 628 16 4·248)

From Antigua to England is remitted 500*l.* What muſt be received exchange, at 135*l.* per cent?

l.		*l.*		*l.*		*l.*
100	:	135	::	500	:	675

How many pounds Sterling muſt be received for 127 guilders, 17 ſtivers, 12 pennings exchange, at 33 *s.* 9 *d.* per pound ?

```
 s.  d.   l.      g.   ſt.  pen.
 33  9  . 1  ::  127   17   12 :
 12               20
----            -----
405             2557
  8               16
----            -----
3240            15344
                2558
               ------  l.   s.   d.
       3240)   40924  (12   12   7 33/81 anſwer.
               3240
               -----
                8524
                6480
               -----
                2044
                  20
               -----
                 &c
```

England remits to France 769 *l.* 17 *s.* how many crowns, &c. must be received at 52 *d.* per crown?

```
d.     c.     l.     s.
52  :  1  ::  769    17
                     20
             ----------
             15397
                12
             ------- crowns  li.  sol.  d.
    52)  184764 (3553    0     9    2 10/13 answer.
         156
         ---
          287
          260
          ---
           276
           260
           ---
            164
            156
            ---
              8
             60
            ---
   52)      480 (9
            468
            ---
             12
             12
            ---
   52)      144 (2
            104
            ---
             40
            ---
```

How many piasters are equal to 510 *l.* Sterling, at 4 *s.* 2 *d.* each? Answer 2448 piasters.

How many pounds Sterling are equal to 2448 piasters at 4 *s.* 2 *d*? Answer 510 *l.*

How

How many milreas, &c. are equal to 213 *l.* 7 *s.* 10 *d.* at 5 *s.* $9\frac{1}{2}$ *d.* per milrea? Anſwer 736 milreas, and $892\frac{111}{125}$ reas.

How many pounds ſterling are equal in value to 736 milreas, and $892\frac{111}{125}$ reas, at 5 *s.* $9\frac{1}{2}$ *d.* per milrea? Anſwer 213 *l.* 7 *s.* 10 *d.*

FELLOWSHIP.

DEFINITION.

108. FEllowſhip determines and adjuſts the gain, loſs, &c. of merchants, trading in company; the equivalent ſhares of creditors in caſes of bankruptcy, &c. and when theſe have no relation to time, it is called *Single Fellowſhip*; but if there is time concerned, then we term it *Double Fellowſhip*, or *Fellowſhip with Time*.

SINGLE FELLOWSHIP.

109. RULE. *As the ſum of the ſtocks, debts,* &c.
Is to the gain, loſs, &c.
So is each particular ſtock, &c.
To the proportional gain, loſs, &c.

Suppoſe *A*'s ſtock in trade be 400 *l.* *B*'s 600 *l.* and *C*'s 900 *l.* and they gain 356 *l.* 10 *s.* what's each man's ſhare thereof?

Now $400 + 600 + 900 = 1900$.

And $1900 : 356{\cdot}5 :: 400 : \frac{356{\cdot}5 \times 400}{1900} = \frac{356{\cdot}5 \times 4}{19} = \frac{1426}{19} =$ 75 *l.* 1 *s.* $0\frac{12}{19}$ *d.* *A*'s gain.

$1900 : 356{\cdot}5 :: 600 : \frac{356{\cdot}5 \times 6}{19} = \frac{2139}{19} =$ 112 *l.* 11 *s.* $6\frac{18}{19}$ *d.* *B*'s gain.

1900

$1900 : 356{\cdot}5 :: 900 : \frac{3208{\cdot}5}{19} = 168\,l.\ 17\,s.\ 4\frac{8}{19}\,d.$ C's gain.

And $75\,l.\ 1\,s.\ 0\frac{12}{19}\,d. + 112\,l.\ 11\,s.\ 6\frac{18}{19}\,d. + 168\,l.\ 17\,s.\ 4\frac{8}{19}\,d. = 356\,l.\ 10\,s.$ proof.

A bankrupt owes to *C* 650 *l.* to *D* 460 *l.* to *E* 350 *l.* to *F* 740 *l.* but his effects are only 1000 *l.* what will it amount to per pound, and how much must each receive?

$$\begin{array}{llllll} & l. & l. & l. & & s.\ \ d. \\ \text{And} & 2200 : & 1000 :: & 1 : \frac{1000}{2200} & = \frac{5}{11} = & 9\ \ 1\frac{1}{11}\ \text{per } l. \end{array}$$

650 Now, seeing $9\,s.\ 1\frac{1}{11}\,d. = \frac{5}{11}\,l.$ is the
460 the money paid, per pound we shall have
350
740

Sum 2200 *l.*

$$\begin{array}{rrrr} & l. & s. & d. \\ \frac{5}{11} \times 650 = & 295 & 9 & 1\frac{1}{11} \\ \frac{5}{11} \times 460 = & 209 & 1 & 9\frac{9}{11} \\ \frac{5}{11} \times 350 = & 159 & 1 & 9\frac{9}{11} \\ \frac{5}{2} \times 740 = & 236 & 7 & 3\frac{3}{11} \\ \hline £.\ & 1000 & & \text{proof.} \end{array}$$

Divide 20 *s.* among 4 persons in proportion as $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$.

$$\left.\begin{array}{l} \frac{1}{3} = \frac{20}{60} \\ \frac{1}{4} = \frac{15}{60} \\ \frac{1}{5} = \frac{12}{60} \\ \frac{1}{6} = \frac{10}{60} \end{array}\right\} \quad \frac{20+15+12+10}{60} = \frac{57}{60}.$$

Now, if the denominator is rejected in the first and third terms, we shall have

$$\left.\begin{array}{rrrrrrrrrrr} & & & & & s. & d. & f. & & \\ 57 & : & 20 & :: & 20 & : 7 & 0 & 0 & 48 & \text{rem.} \\ 57 & : & 20 & :: & 15 & : 5 & 3 & 0 & 36 & \\ 57 & : & 20 & :: & 12 & : 4 & 2 & 2 & 6 & \\ 57 & : & 20 & :: & 10 & : 3 & 6 & 0 & 24 & \\ \hline & & & \text{Proof} & & 20 & & & & \end{array}\right\}\ \text{answers.}$$

Four

Four perſons agree to buy 150 gallons of rum for 22*l.* 10*s.* of which ſum *A* pays 7*l.* 10*s.* *B* 6*l.* 15*s.* *C* 5*l.* 5*s.* and *D* the remainder, how many gallons of rum muſt each have? Anſwer *A* 50 gallons, *B* 45, *C* 35, and *D* 20.

A ſhip worth 900*l.* being entirely loſt, of which $\frac{1}{8}$ belonged to *S*, $\frac{1}{4}$ to *T*, and the reſt to *V*, what loſs will each ſuſtain, ſuppoſing 540 l. of her inſured? Anſwer, *S* will loſe 45*l.* *T* 90*l.* and *V* 225*l.*

FELLOWSHIP *with* Time; *or*, DOUBLE FELLOWSHIP.

110. Rule. *Multiply each man's ſtock by the time of its continuance, uſing their products as the ſtock, &c. in Single Fellowſhip.*

Example.

A, *B*, and *C*, make a ſtock for 18 months, *A* puts in 65*l.* and 5 months after he puts in 40*l.* more; *B* puts in 50*l.* and 6 months after 50*l.* more, but 5 months after this he took all his money out; *C* puts in at firſt 300*l.* but 6 months after took out 250*l.* now, during the 18 months partnerſhip they gained 250*l.* what muſt each have of the profit?

65	65		50	50
5	40		6	50
325	105		300	100
	13			5
	315			500
	105			300
	1365	*B*'s ſtock and time		800
	325			
A's ſtock and time	1690			

300	300	*A*'s 1690
6	250	*B*'s 800
1800	50	*C*'s 2400
	12	Sum 4890
	600	
	1800	
	C's stock and time 2400	

$$\begin{array}{lllll} & l. & & & l. \\ 4890 : 250 :: 1690 : & \frac{1690 \times 25}{489} & = 86{\cdot}4 \\ 489 : 25 :: 800 : & \frac{20000}{489} & = 40{\cdot}899 \\ 489 : 25 :: 4400 : & \frac{25 \times 2400}{489} & = 122{\cdot}7 \\ & \text{Proof} & 249{\cdot}999 \end{array}$$

111. This rule is so little used in business (if any at all) that I judge it unnecessary to give more examples, nor indeed is it just in many cases, for instance, let us suppose *two merchants put into company each* 500l. *and at the end of* 6 *months one of them having occasion for cash, withdraws his* 500l. *tho' the company's accounts cannot be settled till* 12 *months are expired, when they find* 100l. *lost by trade*, now according to the rule, he who took his money out, would only lose 25*l.* whereas the others loss would amount to 75*l.* which is a contradiction to common sense, and the very nature of trade.

The rules of *alligation*, *equation of payments* and *false possition* are omitted, seeing none of them can be well accounted for, without the assistance of algebra, and are of but little use independant thereof.

ALGEBRA.

ALGEBRA.

DEFINITION.

112. ALGEBRA has been well defined by Sir Isaac Newton, an *universal arithmetic*, because of its general and extensive use in all mathematical sciences, and in none more so than in arithmetic, to which we intend to confine our speculations in this treatise. The solution of problems in Algebra, depend upon the following self-evident axioms.

113. AXIOM 1st, *If equal quantities are added to equal quantities, their sums will be equal.*

2d, *If equal quantities are taken from equal quantities, their differences will be equal.*

3d, *If equal quantities are divided by equal quantities, their quotients will be equal.*

4th, *Quantities equal to one and the same quantity, are themselves equal.*

ADDITION *of* ALGEBRAIC QUANTITIES.

114. RULE 1. *If the quantities are alike*, and have the same sign, take the sum of their Coefficients †, to which prefix the common sign, and place the common quantities or letters after.*

EXAMPLES.

* Quantities are said to be alike, when they are composed of the same letters, however the signs and coefficients may differ.

† When the first term is plus or + it is seldom written, though we must be careful to understand it before such quantities.

EXAMPLES.

To	a	$-4x$	$4y$	*$12a$	$-5bcd$	axy
Add	$7a$	$-6x$	$8y$	$6a$	$-7bcd$	axy
Sum	$8a$	$-10x$	$12y$	$18a$	$-12bcd$	$2axy$

115. RULE 2. *If like quantities have unlike signs, take the difference of the coefficients, to which prefix the sign of the greater quantity, and place the common letters or quantity after the said difference.*

EXAMPLES.

To	$-5acd$	$4xy$	$5x-4y+6aa$	$-axy+3azy$
Add	$3acd$	$-xy$	$-7x-6y-8aa$	$2axy-9azy$
Sum	$-2acd$	$3xy$	$-2x-10y-2aa$	$-axy-6azy$

116. RULE 3. *To add quantities which are unlike, set them all down one after another, connecting them with their proper signs.*

EXAMPLES.

To	$5ax+ay-z$
Add	$7az-ab+z$
	$5ax+ay+7az-ab$

To	$7azz-axy+56$
Add	$7az-ax+5a$
	$7azz-axy+56-7az-ax+5a$

RULE 4. *When quantities are so compounded as to have like and unlike signs, and like and unlike quantities, collect them into the least compass by the three forgoing Rules.*

* The numbers prefixed to any quantity are termed *coefficients*, and when there are no numbers before a quantity, we understand unity as its coefficient: Thus a and $1a$ are equal.

EXAMPLES.

To	$7ax - az + 5ax$	To	$7a + 5aa - 3ay$
Add	$ay + 3az + 7ax$	Add	$-5a + 7ay - 6aa$
Sum	$19ax + 2az + ay$	Sum	$2a - aa + 4ay$

$$\text{Add}\begin{cases} 7az + 5ay - x \\ -5az + 7ay - xy \\ 4ay - 3ax + 5x \\ 3ay - 6ax + xy^{*} \end{cases}$$

$$\text{Sum } 2az + 19ay - 4x - 9ax$$

SUBTRACTION *of* ALGEBRAIC QUANTITIES.

118. RULE. *Change all the signs in the subtrahend, (or number to be subtracted) or suppose them changed, and then proceed by the rules in Addition.*

From $7ax - ay + az$ take $-5ax - ay - az$

$5ax + ay + az$ subtrahend with the signs changed†

Diff. $12ax + 2az$

From $7ayz - xy + 76$ take $7ayz - 3xz + 100$

$-7ayz + 3xz - 100$

Diff. $xy + 3xz - 24$

From	$5xxyy - 6byy + axy$
Take	$xxyy - 5byy - 567$ ‡
Diff.	$4xxyy - byy + axy - 567$

MUL-

* Because $-xy$ and $+xy$ are equal quantities with contrary signs, and by consequence of contrary values they destroy each other, or their sum is 0; for should a person's effects be expressed by $+10l.$ and his debts by $-10l.$ he is upon the whole worth nothing; and hence it is, that the signs $+$ and $-$, are termed affirmative and negative respectively.

† Because the signs $+$ and $-$ are contrary in their signification, and the rules of Addition and Subtraction are so likewise; therefore 'tis manifast that if the value of the subtrahend is changed by changing the signs, the work ought to be performed by Addition.

‡ In this example the signs are not actually changed, but they are carefully to be considered as such in the operation.

MULTIPLICATION of ALGEBRAIC QUANTITIES.

119. RULE 1. *Multiply the coefficients of ſimple quantities together, and to the product annex the quantities in both factors; if the ſigns be alike the product is affirmative or +, but if unlike negative or —.*

EXAMPLES.

Multiply	$5ax$ *	$-7ay$ †	$-a$ ‡
By	$6ay$	xx	$-x$
	$30aaxy$	$-7ayxx$	ax

Multiply a, b, c, together.

here $a \times b \times c = abc$ Anſwer. or

$$\begin{array}{l} a \\ b \\ \hline ab \\ c \\ \hline abc \text{ Anſ.} \end{array}$$

120. RULE 2. *A compound quantity is multiplied with a ſingle quantity, by multiplying it into every member of the compound quantity, and connecting the product by their proper ſigns.*

 EXAM-

* When an affirmative quantity as $+5ax$ is multiplied by another affirmative one, as $+6ay$, the meaning is that $+5ax$ is to be taken ſo many times as there are units in $6ay$; the product is therefore evidently $6ay$ times $5ax$, or $30aaxy$.

† If $-7ay$ is multiplied by xx, then is $-7ay$ to be taken ſo many times as there are unites in xx, and the product is evidently xx times $-7ay$ or $-7ayxx$.

‡ When $-a$ is multiplied by $-x$, then is $-a$ to be ſubtracted as often as there are units contained in x; for if multiplication by an affirmative number is a repeated addition, it muſt by a negative one be a repeated ſubtraction, but to ſubtract $-a$ is the ſame thing as adding $+a$; therefore the product is really $+ax$.

EXAMPLES.

Multiply $5ax - ay + cd$
By $5xx$

Product $25axxx - 5ayxx + 5cdxx$

Multiply $5ax + vy - 5da$
By $-a$

$-5aax - avy + 5daa$

RULE 3. *To multiply quantities in which both factors are compound; multiply every member of the multiplier into every member in the multiplicand, having due regard to the signs, and collect the several products into a sum by Addition.*

EXAMPLES.

Multiply $ax + ay$
By $2x + y$

$2axx + 2axy$
$axy + ayy$

Product $2axx + 3axy + ayy$

Multiply $x - y$
By $x + y$

$xx - xy$
$xy - yy$

Product $xx - yy$

Mul-

Multiply	$x+5a$
By	$5y+7a$
	$5xy+25ay$
	$7ax+35aa$
Product	$5xy+25ay+7ax+35aa$

Multiply	$ax+ay+zx$
By	$ax-ay+xy$
	$aaxx+aayx+azxx$
	$-aayx-aayy-ayzx$
	$ayxx+axyy+zyxx$
	$aaxx+azxx-aayy-ayzx+ayxx+axyy+zyxx$

DIVISION *of* ALGEBRAIC QUANTITIES.

121. RULE. *The signs are managed by the rules in Multiplication, and the work performed by considering what the quotient must be, so that, multiplied by the divisor, it may produce the dividend.*

EXAMPLES.

Divisor	Dividend	Quotient		
$6ay)$	$30aayx$	$(5ax$	$-7ay)$	$-7ayxx\ (xx$
	$30aayx$			$-7ayxx$

$$xx)\ -7ayxx\ (-7ay$$
$$-7ayxx$$
$$*\ *$$

 $-a)$

$$\begin{array}{l} -a)\ -5aax-avy+5daa\ (5ax+vy-5da \\ \quad\ -5aax \\ \hline \qquad\qquad -avy \\ \qquad\qquad -avy \\ \hline \qquad\qquad\qquad 5daa \\ \qquad\qquad\qquad 5daa \\ \hline \qquad\qquad\qquad * \ * \end{array}$$

$$\begin{array}{l} x-y)\ xx-2xy+yy\ (x-y \\ \quad\ xx-xy \\ \hline \qquad -xy+yy \\ \qquad -xy+yy \\ \hline \qquad\quad * \quad * \end{array}$$

Of ALGEBRAIC FRACTIONS.

122. FRactions in Algebra are managed in the same manner as those in whole numbers, and therefore we need not repeat the rules, but proceed to

EXAMPLES.

Reduce $\frac{ay+xy}{yd}$ to its lowest terms.

If every member of this fraction be divided by y, we shall have $\frac{ay+xy}{yd}=\frac{a+x}{d}$; and therefore when it happens that the same letter or quantity will divide every term in each factor, such a quantity may be expunged per art. 59. Thus $\frac{15ax-12ay-9a}{3a}=5x-4y-3$.

Reduce $\frac{5a}{3}$ and $\frac{a+y}{x+y}$ to a common denominator.

By

By art. 82, we get $\left\{\begin{array}{l} 5a \times \overline{x+y} = 5ax+5ay \\ 3 \times \overline{a+y} = 3a+3y \end{array}\right\}$ numer.

$3 \times \overline{x+y} = 3x \times 3y$ the denom.

Hence $5\dfrac{ax+5ay+3a+3y}{3x+3y}$ is the anſwer.

To $\dfrac{5a}{3}$ add $\dfrac{7ax}{2}$.

$\left.\begin{array}{l} 5a \times 2 = 10a \\ 7ax \times 3 = 21ax \end{array}\right\}$ $3 \times 2 = 6$ hence the anſwer is

$\dfrac{10a+21ax}{6}$.

Let $ay + \dfrac{3a}{y}$ be multiplied by $\dfrac{ax}{z} - y$.

$$ay + \frac{3a}{y}$$
$$\frac{ax}{z} - y$$

$$\frac{aayx}{z} + \frac{3xaa}{z}$$
$$-ayy - 3a$$

$$\frac{aavx}{z} + \frac{3aax}{z} - ayy - 3a \text{ anſwer.}$$

Divide $\dfrac{5ax}{7}$ by $\dfrac{5}{y}$

Per art. 87, $\dfrac{y}{5} \times \dfrac{5ax}{7} = \dfrac{5axy}{35} = \dfrac{axy}{7}$ the anſwer.

Any perſon ſkilled in Vulgar Fractions, will eaſily perform examples of this kind, in the Algebraic form.

Of

Of INFINITE SERIES.

123. WHEN it happens that the dividend does not contain the divisor an equal number of times, the quotient may be continued in an infinite series.

EXAMPLES.

$$
\begin{array}{llllll}
1-x\,)\;1 & & & & (\,1+x+xx+xxx,\ \&c.^{*} \\
\quad\;\; 1-x & & & & \\
\hline
 & x & & & \\
 & x-xx & & & \\
\hline
 & & xx & & \\
 & & xx-xxx & & \\
\hline
 & & & xxx & \\
 & & & xxx-xxxx & \\
\hline
 & & & & xxxx
\end{array}
$$

* It is easy to observe, without proceeding further, in what manner the terms of this quotient do arise, which is called *discovering the law of the series*.

Again

Again, if it be required to divide ee by $e-a$, we shall have

$$
\begin{array}{l}
e-a\,)\,ee \qquad (\,e+a+\frac{aa}{e}+\frac{aaa}{ee}+\frac{aaaa}{eee}, \text{ \&c.} \\
\quad ee-ae \\
\hline
\qquad\quad ae \\
\qquad\quad ae-aa \\
\hline
\qquad\qquad\quad aa \\
\qquad\qquad\quad aa-\frac{aaa}{e} \\
\hline
\qquad\qquad\qquad\quad \frac{aaa}{e} \\
\qquad\qquad\qquad\quad \frac{aaa}{e}-\frac{aaaa}{ee} \\
\hline
\qquad\qquad\qquad\qquad\qquad \frac{aaaa}{ee}, \text{ \&c.}
\end{array}
$$

Alſo $\frac{ex}{e-s}$ will be found $= x+\frac{x}{e}+\frac{x}{ee}+\frac{x}{eee}$, &c.

and $\frac{aa+yy}{a+y}=a-y+\frac{2yy}{a}-\frac{2yyy}{aa}+\frac{2yyyy}{aaa}$, &c.

where you will find the terms which ariſe are affirmative and negative alternately. The method of expreſſing fractions in infinite ſeries, will be of ſingular uſe to the learner, in many braaches of the Mathematics.

INVOLUTION.

124. IS the multiplying any quantity continually by itſelf, and the ſeveral products thence ariſing are called the ſquare, cube, biquadrate, &c. otherwiſe the 2d, 3d, 4th, &c. powers; the quantity itſelf being the root or firſt power, all which is plain from the annexed table.

$a =$

a = the root or firſt power
$a \times a = aa$ = the ſquare or 2d power
$a \times a \times a = aaa$ = the cube or 3d power
$a \times a \times a \times a = aaaa$ = the biquadrate or 4th power
$a \times a \times a \times a \times a = aaaaa$ = the furſolid or 5th power

125. We uſually write the quantity thus involved only once, with a figure called the exponent or index above it, denoting the height to which it is raiſed: thus aa we expreſs a^2, $aaa = a^3$, $aaaa = a^4$, &c.

Examples.

Involve $bbaaa$ or $b^2\,a^3$ to the 2d power or ſquare.

$bbaaa \times bbaaa = bbbbaaaaaa = b^4\,a^6$ anſwer.

Involve $-2ax$ to the 4th power.

$$\begin{array}{r} -2ax \\ -2ax \\ \hline 4a^2x^2 \\ -2ax \\ \hline -8a^3x^3 \\ -2ax \\ \hline 16a^4x^4\ * \\ \hline \end{array}$$

126. Hence we obſerve, that ſimple quantities are involved to any deſired power, by multiplying the index of the given quantity, by the digit expreſſing the height of the required power, and the co-efficients (when there are any) to the power which the ſaid index directs. Thus

* It appears by the involution of a negative quantity, that the ſigns of the ſeveral powers are affirmative and negative alternately, viz. affirmative when the index of the power is even, but negative when odd.

Thus if we ſquare x^2 we have $x^{2\times 2} = x^4$ and x cubed is $x^{1\times 3} = x^3$ alſo if we involve $x^2 \; y^3$ to the 9th power, it will be $x^{2\times 9}\, y^{3\times 9} = x^{18}\, y^{27}$. Again, if the ſquare of $9\, xy^3\, z^3$ is required, it will be $9 \times 9 \times x^{1\times 2}\, y^{2\times 2}\, z^{3\times 2} = 81\, x^2\, y^4\, z^6$.

127. It is likewiſe not difficult to diſcover, that ſuch quantities may be multiplied and divided by adding and ſubtracting their indices or exponents. Thus if a^3 is multiplied by a^2, the product will be $a^{3+2} = a^5$; alſo $6^2\, a^3 \times 6^3\, a^2$ will be $6^5\, a^5$ and $\frac{a^6}{a^3} = a^{6-3} = a^3$; alſo $\frac{a^9}{ba^3} = \frac{a^6}{b}$. But if the index of the diviſor be the greater quantity, the index of the quotient will be negative; for $\frac{a^2}{a^3} = a^{2-3} = a^{-1}$; alſo $\frac{x^7}{x^9} = x^{7-9} = x^{-2}$. Now if the laſt fraction, viz. $\frac{x^7}{x^9} = \frac{xxxxxxx}{xxxxxxxxx}$, be reduced to the loweſt terms, it is evidently $\frac{1}{xx} = \frac{1}{x^2}$, which is equal x^{-2}.

128. Let x^{-2} and x^{-3} be multiplied together, and we ſhall have $x^{-2+-3} = x^{-5}$; or, which is better, take their equals, viz. $\frac{1}{x^2} \times \frac{1}{x^3}$, and we have $\frac{1}{x^5}$. Again let the quotient of $\frac{x^{-5}}{x^{-7}}$ be required, now by taking their equals $\frac{1}{x^5}$ and $\frac{1}{x^7}$ and inverting the diviſor they ſtand

ſtand $\frac{1}{x^5} \times \frac{x^7}{1} = \frac{x^7}{x^5} = x^2$; and hence, obſerve in general, *that any quantity in the denominator may be written in the numerator, and the contrary, by changing the ſign of the index.*

129. If m and n denote the exponents of x in general, their product will be $x^m \times x^n$ and their quotient $\frac{x^m}{x} = x^{m-n}$

130. The method of involving compound quantities is no other than a continual multiplication by the root, to the height the index of the required power directs.

EXAMPLES.

Involve $x+y$ called abinomial to the 4th power.

$$
\begin{array}{llll}
x+y & & & \text{the root or 1ſt power.} \\
x+y & & & \\
\hline
x^2+xy & & & \\
\quad xy+y^2 & & & \\
\hline
x^2+2xy+y^2 & & & \text{the 2d power.} \\
a+y & & & \\
\hline
x^3+2x^2y+xy^2 & & & \\
\quad x^2y+2xy^2+y^3 & & & \\
\hline
x^3+3x^2y+3xy^2+y^3 & & & \text{the 3d power.} \\
x+y & & & \\
\hline
x^4+3x^3y+3x^2y^2+xy^3 & & & \\
\quad x^3y+3x^2y^2+3xy^3+y^4 & & & \\
\hline
x^4+4x^3y+6x^2y^2+4xy^3+y^4 & & & \text{the 4th power.} \\
\hline
\end{array}
$$

Involve

Involve $x-y$ termed a recidual to the 4th power.

$$\begin{array}{l} x-y \\ x-y \\ \hline x^2-xy \\ \quad -xy+y^2 \\ \hline x^2-2xy+y^2 \text{ the 2d power.} \\ x-y \\ \hline x^3-2x^2y+xy^2 \\ \qquad x^2y+2xy^2-y^3 \\ \hline x^3-3x^2y+3xy^2-y^3 \text{ the 3d power.} \\ x-y \\ \hline x^4-3x^3y+3x^2y^2-xy^3 \\ \quad -x^3y+3x^2y^2-3xy^3+y^4 \\ \hline x^4-4x^3y+6x^2y^2-4xy^3+y^4 \text{ the 4th p.} \end{array}$$

131. Hence we discover, 1st, That the terms arising from the involution of $x+y$ and $x-y$ are the same, but that the signs of the latter are $+$ and $-$ alternately. 2dly, That the sum of the exponents of x and y in every term is the same, and that sum equal the power involved to. 3dly, That the first term of any power is the quantity x raised to the said power. 4thly, That the powers of x decrease in arithmetical progression from the first term till its index be $x^0 = 1$. 5thly, That the exponents of y arise in the same manner as those of x decrease.

132. We also observe: 1st, When the number of terms is odd, the co-efficient of the middle term is greatest. 2dly, When the number of terms is even, the two middle terms have their co-efficients equal and greatest also. 3dly, That the co-efficients at equal distances from the middle terms, are equal, and that they decrease on each side the middle terms in the same regulary order to unity. And hence we get the following general rule to find the co-efficients to any power.

Rule. *Multiply the exponent of* x *in the preceeding term by its co-efficient, and divide the product by the exponent of* y *in the given term, and the quotient is the required co-efficient.*

133. Let it be required to involve $x - y$ to the 7th power: By art. 131, the terms without the co efficients are, $x^7 - x^6y + x^5y^2 - x^4y^3 + x^3y^4 - x^2y^5 + xy^6 - y^7$ and the co-efficients will be found per rule as follows: $\frac{7 \times 1}{1} = 7$. $\frac{6 \times 7}{2} = 21$. $\frac{21 \times 5}{3} = 35$. &c. hence we get $x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$ for the anſwer.

134. But generally let $x + y$ be involved to the mth power. The terms without the co-efficients will ſtand thus $x^m + x^{m-1}y + x^{m-2}y^2 + x^{m-3}y^3 + x^{m-4}y^4$ &c. continued till the exponent of y be equal m, for then muſt the index of x be $= 0$. The co-efficients are by the rule article 132, as under, viz. $1.\ m.\ \frac{m-1}{2} \times m.\ \frac{m-1}{2} \times m \times \frac{m-2}{3}.\ \frac{m-1}{2} \times m \times \frac{m-2}{3} \times \frac{m-3}{4}$, and ſo on till the number of co-efficients found exceed the units contained in m by 1, and hence $x^m + mx^{m-1}y + \frac{m-1}{2} \times mx^{m-2}y^2$ &c. is the terms of the required ſeries.

EVOLUTION.

135. Evolution is the reverſe of involution, and therefore requires a contrary proceſs, viz. *to divide the exponents of the given quantity by the index of the required root.* Thus the ſquare root of x^2 is $x^{2 \div 2} = x$ of x^4 it is $x^{4 \div 2} = x^2$, alſo the ſquare root of x^2y^2 is xy, and of x^4y^6 it is x^2y^3, and the cube root of x^3 is x of x^3y^3 it is xy, and of $x^{12}y^3a^6$ it is x^4ya^2; but the ſquare

ſquare root of x^3 is $x^{\frac{3}{2}}$ of $x^5 = x^{\frac{5}{2}}$, alſo of x it is $x^{\frac{1}{2}}$ and the cube root of x^2 is $x^{\frac{2}{3}}$ of x^3y^4 it is $xy^{\frac{4}{3}}$. The ſigns are managed by the rules delivered in involution.

136. And hence we find, that the power of an affirmative quantity, may have a negative root, for the ſquare root of a^2, may be either $+ a$ or $- a$, becauſe $a \times a = a^2$ and $-a \times -a = a^2$ likewiſe.

137. From which we infer, that the root of a negative quantity cannot be taken, if the index of the ſaid root be an even number, thus ſhould the ſquare of $-a^2$ be required, it can be neither a nor $-a$, ſeeing the ſquare of either of them is a^2.

138. But when the root is denoted by an odd number, the ſigns are always the ſame as that of the power, for the cube root of $-a^3$ is $-a$ of a^3 it is a, &c.

139. When the index of the root will not divide the index of the power without a remainder, ſuch are termed ſurds; thus the ſquare root of x^3 is $x^{\frac{3}{2}} = \sqrt{a^3}$ and of x^5 it is $\sqrt{x^5}$, the cube root of a^2 is $a^{\frac{2}{3}} = \sqrt[3]{a^2}$, alſo if the nth power of x^m be required, it is $\sqrt[n]{x^m}$ or $x^{\frac{m}{n}}$: theſe imperfect powers or ſurds are multiplied, divided, involved, &c. in the ſame maner as others, viz. by the addition, ſubſtraction, &c. of their exponents.

140. As to the roots of compound quantities, they cannot be taken in finite terms, unleſs their powers are ſuch as can be diſcovered to have ariſen from ſome known root. For inſtance, the ſquare of $x + y$ conſiſts of three terms, viz. 1ſt, The ſquare of x. 2dly, The product of xy multiplied by 2. 3dly, The ſquare of y; thus, $x^2 + 2xy + y^2$ is the power and $x + y$ the ſquare root. Alſo the ſquare root of $x^2 - 2xy + y^2$ is $x - y$

$x-y$ and the ſquare root of $x^2+xy+\frac{y^2}{4}$ is $x+\frac{y}{2}$.
Any root of compound quantities may be aproximated to any aſſigned degree of exactneſs by the general formula, delivered article 134, for m may repreſent any quantity, either a whole number or a fraction.

The SQUARE ROOT.

DEFINITION.

141. TO extract the ſquare root of any numerical quantity, is to find ſuch a number, as being multiplied by itſelf, will produce the given quantity.

142. Becauſe any ſingle digit, multiplied by itſelf, can produce only two digits in the product, the given quantities are reſolved into periods of two ſigures each, which denote the number of digits the required root will conſiſt of: thus the quantity 784687 is reſolved into three periods by the comma points, conſiſting of two figures each, and 8765765 being an odd number of digits, conſiſts of two figures to each alſo, only the left hand digit, becauſe odd is itſelf a period.

143. Hence the ſquare root of any quantity, conſiſting of two figures only, may be known from the common multiplication table: thus the ſquare root of 81 is 9, for 9×9 is 81, agreeable to the definition by the ſame manner of reaſoning, is the root of any ſingle period diſcovered; but if 625 is given, the root contains two digits. Let that in the place of tens be x, and that in the units place y; then, if $x+y$ is ſquared, we get $x^2+2xy+y^2$, which by the ſuppoſition is equal to 625.

There-

$$
\begin{array}{rrl}
\text{Therefore} & 625 & (20 = x \\
x^2 = & 400 & \\
\hline
& 225 & (\ 5 = y \\
\text{then } 2xy + yy = \overline{40 + 5} \times 5 = & 225 & \\
\hline
& & \text{Sum } 25 = x + y
\end{array}
$$

Again let the ſquare root of 219024 be required, and as there are three periods, let the root be $x + y + z$, then is the ſquare thereof per involution $= x^2 + 2xy + y^2 + 2zx + 2yz + z^2 = 219024$

$$
\begin{array}{rrl}
\text{And} & 219024 & (400 = x \\
x^2 = & 160000 & \\
\hline
& 59024 & (\ 60 = y \\
\text{and } 2xy + yy = \overline{2x + y} \times y = \left.\begin{array}{r} \\ 800 + 60 \times 60 = \end{array}\right\} & 51600 & \\
\hline
& 7424 & (\ \ 8 = z \\
\text{again } 2xz + 2yz + z^2 = \overline{2x + 2y + z} \left.\begin{array}{r} \\ \times z = \overline{800 + 120 + 8} \times 8 = \end{array}\right\} & 7424 & \\
\hline
& & 468 \text{ anſwer.}
\end{array}
$$

Omitting the cyphers in the preceding example, the work will ſtand thus.

$$
\begin{array}{rl}
& 219024 \ (468 \text{ anſwer.} \\
& 16 \\
\hline
86) & 590 \\
& 516 \\
\hline
928) & 7424 \\
& 7424 \\
\hline
\end{array}
$$

144. Hence the following general rule is deduced.

RULE. *Having pointed the giving quantity as before directed, take the greatest root in the first period, and subtract its square from the said period; bring down the the next pair of figures to the remainder for a new dividend; double the root for a divisor; ask how often this divisor is contained in the dividend; place the digit expressing the number of times both in the root and in the units place of the divisor; then multiply and subtract as in common division; proceeding in like manner till the figures in the dividend are all brought down, and if necessary, annex pairs of cyphers to the dividend, with which proceed as before, and you will get decimal places to the root, which may be carried forward to any required degree of exactness.*

EXAMPLES.

Required the square root of 87467.

```
          , , ,
        ) 87467 (295,74 * &c.
          4
         ----
   49)   474
         441
         ----
  585)    3367
          2925
          ----
 5907)     44200
           41349
          ------
59144)     285100
           236576
          -------
            48524 &c.
          -------
```

Re-

* Such quantities, as the above, will never come out without a remainder, and they are for this reason termed surds or irrational quantities.

Required the square root of ·2.

```
   , , ,
   200000 (·447, &c.
   16
   ———
84) 400
    336
   ———
887) 6400
     6209
    ————
      191 &c.
    ————
```

What is the square root of 4857532416. Ans. 69696.

Required the square root of ·00015625. Ans. ·0125.

What is the square root of 2. Ans. 1·4142135, &c.

Required the square root of $\frac{9}{16}$.* Ans. $\frac{3}{4}$.

The CUBE ROOT.

DEFINITION.

145. TO extract the cube root, is to find such a number, which if multiplied by itself, and the product multiplied again by the said number, shall, in the last product, produce that quantity the root of which was to be extracted.

146. For the same reason that the given quantities in the square root are resolved into periods of two digits each, those in the cube root will contains three, and where the quantity proposed contains but a single period, the root will be easily found; for if we require the

* When the square root of a vulgar fraction is required, 'tis found by taking the root of both factors.

the cube root of 729, it will by 9, becaufe $9 \times 9 \times 9 = 729$: But if the cube root of 13824 is required, the root will confift of two figures. Let that in the place of tens be x and the other y, then will the cube of $x + y$ be $x^3 + 3x^2y + 3xy^2 + y^3 = 13824$.

$$\begin{array}{rrl} \text{And} & 13824 & (20 = x \\ x^3 = & 8000 & \\ \hline & 5824 & (\ 4 = y \\ \hline 3x^2y = & 4800 & \\ 3xy^2 = & 960 & 24 \text{ anfwer.} \\ y^3 = & 64 & \\ \hline 3x^2y + 3xy^2 + y^3 = & 5824 & \\ \hline \end{array}$$

147. From hence we derive the following general

RULE. *Having pointed the given quantity as directed, take the greateft root in the firft period, and fubtract its cube from the faid period, and to the remainder, bring down the next period, which call the dividend; then call the quotient figure, with a cypher annexed thereto,* x; *put three times* x^2 *for a divifor, and call the number of times that* $3x^2$ *is contained in your dividend* y, *which place in the root or quotient; then find the values of* $3x^2y$, $3xy^2$ *and* y^3 *the fum of which take from what you called the dividend to the remainder; bring down another period, and call the figures in the root, with a cypher anneaed,* x, *proceeding in the fame manner as before, till all the periods are brought down, and if decimals are required in the root, bring down periods of cyphers to the remainder, which will give the root to any exactnefs required.*

Required the cube root of 51686·703125.

$$
\begin{array}{rr}
 & \overset{,}{5}1\overset{,}{6}86\cdot\overset{,}{7}03\overset{,}{1}25\ (3725 \\
 & 27 \\
\hline
\left.\begin{array}{r} 30 = x \\ 7 = y \end{array}\right\} 3x^2 = 2700) & 24686 \\
\hline
3x^2y = & 18900 \\
3xy^2 = & 4410 \\
y^3 = & 343 \\
\hline
 & 23653 \\
\hline
\text{2dly, } \left.\begin{array}{r} 370 = x \\ 2 = y \end{array}\right\} 3x^2 = 410700) & 1033703 \\
\hline
3x^2y = & 821400 \\
3xy^2 = & 4440 \\
y^3 = & 8 \\
\hline
 & 825848 \\
\hline
\text{3dly, } \left.\begin{array}{r} 3720 = x \\ 5 = y \end{array}\right\} 3x^2 = 41515200) & 207855125 \\
\hline
3x^2y = & 207576000 \\
3xy^2 = & 279000 \\
y^3 = & 125 \\
\hline
 & 207855125 \\
\hline
\end{array}
$$

What's the cube root of 673373097125. Anſ. 8765.

What's the cube root of $\frac{27}{64}$. Anſ. $\frac{3}{4}$.

148. The biquadrate root is had by extracting the ſquare root of the given quantity, and again the ſquare root

root of that root, for the ſquare root of a^4 is a^2, and the ſquare root of a^2 is a, and the biquadrate root of a^4 is $a^{4 \div 4} = a$. The root of any power may be had from comparing it with the formula of that power.

Of SURDS.

149. THE addition, ſubtraction, multiplication, &c. of ſurds are performed by the ſame rules as rational quantities; we ſhall therefore give a few examples promiſcuouſly, for the better illuſtration of their management.

$$\begin{array}{rl} \text{To} & 5ax\sqrt{xx} \\ \text{add} & 2\sqrt{xx} \\ \hline \text{ſum} & 5ax\sqrt{xx} + 2\sqrt{xx} \end{array}$$

$$\begin{array}{rl} \text{To} & -5x + 6ax\sqrt{x+xx} \\ \text{add} & 7x + ax\sqrt{x+xx} \\ \hline & 2x + 7ax\sqrt{x+xx} \text{ Anſwer.} \end{array}$$

$$\begin{array}{rl} \text{From} & 5ab + \sqrt{xx+yy} \\ \text{take} & 7ab - \sqrt{xx+yy} \\ \hline \text{diff.} & -2ab + 2\sqrt{xx+yy} \end{array}$$

$$\begin{array}{rl} \text{From} & a\sqrt{xx+a} \\ \text{take} & x\sqrt{xx+a} \\ \hline \text{diff.} & a\sqrt{xx+a} - x\sqrt{xx+a} \end{array}$$

Multiply

Multiply $x\sqrt{xx-yy}$
by $x\sqrt{xx-yy}$

product $xx \times \overline{xx-yy} = x^4 - x^2y^4$

Multiply $xx+\sqrt{xx-yy}$
by $x+\sqrt{ax}$

$x^3 + x\sqrt{xx-yy}$
$xx\sqrt{ax}+\sqrt{axxx-axyy}$

product $x^3 + x\sqrt{xx-yy} + xx\sqrt{ax} + \sqrt{ax^3 - axy^2}$

Involve $\sqrt{xx+yy}$ to the square.*
Answer $xx+yy$

Involve $\sqrt[3]{\dfrac{xx+yy}{6ax}} \times a$ to the cube.

Answer $\dfrac{xx+yy}{6ax} \times a^3 = \dfrac{xxa^2+y^2a^2}{6x}$

Of EQUATIONS.

150. EQUATIONS are the expressions of equality between quantities, and are ordered by the following rules.

151. Quantities are transposed from one side of an equation to the other, by changing the sign. (See art. 101, axiom 1st and 2d.)

152.

* Seeing the index of this surd character denotes, that the quantity to which it is prefixed, is to have the square root extracted, therefore involving it to the square, is done by writing the quantity without the sign.

152. If there be fractions in an equation, the denominator will be taken away, by multiplying all the quantities in the equation by it. (Per. art. 101 axiom 3d.)

153. Any quantity will be taken away from a given quantity, by dividing the whole equation by the quantity you would take away. (Art. 101, axiom 4th.)

154. If one side of an equation be contained in a surd, it will be taken away by involving both sides to the height the index directs. (Per. art. 101, axiom 3d.)

155. If one side of an equation contain some power of an unknown quantity, the root will be discovered by evolving both sides agreeable to the index of the power. (Per art. 101, axiom 4th.)

156. If there are too unknown quantities, there must be two equations arising from the data and conditions of the question: and the value of one of those quantities, in both equations, must be found, which being equal, will form a new equation, in which there are but one unknown quantity, which will be solved by the foregoing rules: and in general there must be as many equations as there are unknown quantities; for should the unknown quantities exceed the equations, the answers will be infinite; and if the equations be more than the unknown quantities, the question may be impossible.

ALGE-

ALGEBRAIC QUESTIONS, *producing simple* EQUATIONS.

Queſt, 1ſt. WHAT number is that to which, if 16 be added, and the ſum multiplied by 5, the the product may be 320?

Let	1	x be the required number;
then	2	$\overline{x+16} \times 5 = 320$ per Queſtion,
that is	3	$5x + 80 = 320$;
by tranſpoſing 80 (per art. 153)	4	$5x = 240$,
which ÷ed by 5 (art. 155.)	5	$x = 48$ the anſwer.

For $48 + 16 = 64$, and $64 \times 5 = 320$ proof.

Queſt. 2d. A farm, conſiſting of 125 acres, is let for 38*l.* 5*s.* the lands conſiſts of two ſorts, the better is at 7*s.* 6*d.* and the worſe at 3*s.* 9*d.* per acre, how many acres of each ſort are there?

Let 38*l.* 5*s.* = 9180 pence be put $= d$, 7*s.* 6 = 90 pence $= 2c$; then will 3*s.* 9*d.* = 45 pence be $= c$; let $a = 125$ the acres of land, and put $x =$ to the quantity of the beſt land, then will $a - x =$ the quantity of the worſe ſort, and $x \times 2c = 2cx$ the value of the beſt land, alſo $\overline{a - x} \times c = ac - cx$ the value of the worſe: The ſum of thoſe two values, viz, $ac + xc = d$ is the value of the whole ground, and if ca be taken from both ſides of this equation, it will become $cx = d - ac$, which divided by c, give $x = \frac{d - ac}{c} = 79$ the acres of the better kind, and therefore $a - x = 46$ the acres of the worſe kind; that the above concluſions are right, is thus proved, 79 acres multiplied by 90 pence is 7110 pence, and 46 acres by 45 pence make 2070 pence, and 7110 + 2070 is 9180 pence = to 38*l.* 5*s.*

Queſt. 3d. A Gentleman has an orchard of fruit trees, $\frac{1}{2}$ bearing apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plumbs, and 50 of them cherries, how many fruit trees has he?

Let x repreſent the number of trees, then we evi-

dently have — —	1	$\frac{x}{2} + \frac{x}{4} + \frac{x}{6} + 15 = x$
1ſt ×ed by 6 — —	2	$3x + \frac{3x}{2} + x + 300 = 6x$
2d ×ed by 2 — —	3	$6x + 3x + 2x + 600 = 12x$
by taking the ſum of the co-effi. of x it is	4	$11x + 600 = 12x$
4 tranſpoſe $11x$ —	5	$602 = x$ the anſwer.

Now $\frac{x}{2} = 300$ apple trees,

$\frac{x}{4} = 150$ pear trees,

$\frac{x}{6} = 100$ plumb tree,

50 cherry trees,

Sum $= 600$ proof.

Queſt. 4th. There is a rod of iron a yard long, at the ends of which hang two weights, viz. one of 15 *lb.* the other of 1 *lb.* query, that point of the rod, where theſe weights being ſuſpended, ſhall be an equilibrium?

Let 36 inches = 1 yard be denoted by a, the greater weight = 15 *lb.* = b, the leſſer = 1 *lb.* = d; then if x be put for the diſtance of the weight b from the point of ſuſpenſion, the diſtance of the weight d will be expreſſed by $a - x$, and becauſe theſe diſtances are reciprocally as the weights from the ſaid point, we therefore get the following analogy, as $d : x :: b : \frac{dx}{b} = a - x$ this equation, multiplied by b, in order to take away the fraction, becomes $dx = ba - bx$, by tranſpoſing

bx

bx we have $dx + bx = ba$, and dividing this laſt expreſſion by the co-efficients of x, that is, by $d + b$, we have the value of $x = \dfrac{ab}{d+b} = 2\frac{1}{4}$ inches and conſequently $a - x = 33\frac{3}{4}$ inches: now if the expreſſion $dx + bx = ba$, is turned into an analogy, we get the following, as $d + b : a :: b : x$, and hence we have this general

RULE. *As the ſum of the weights is to the length of the beam or rod, ſo is the leſs weight to the diſtance of the greater from the required point or fulcurum.*

By this rule, may a very uſeful inſtrument the *Steel Yard* be conſtructed.

Queſt. 5th. There are two numbers, the ſum of which is 240, and the greater to the leſs in the ratio of 7 to 3, query the ſaid numbers?

Put	1	$x =$ the greater number
and	2	$240 - x =$ the leſs
then per queſt.	3	$x : 240 - x :: 7 : 3$
and $\times$ing extreams and means (art. 59)	4	$3x = 1680 - 7x$
4th $+ 7x$*	5	$10x = 1680$
5th $\div$ 10	6	$x = 168$ the greater number
per 2d ſtep	7	$240 - x = 72$ the leſs,

and $168 : 72 :: 7 : 3$ the proof.

Queſt. 6th. Three gameſters, *A*, *B*, *C*, being at a table, *A* had 220 before him, *B* 176 and *C* 154: but on a conſtable's coming, each endeavoured to take his money; but in their hurry mixed it, each man taking as much as he could get. It afterwards appeared, that if *A* had laid down $\frac{3}{4}$ of what he caught, *B* $\frac{1}{2}$, and *C* $\frac{1}{4}$, and then dividing the ſum laid down into three equal parcels, then each man had his right ſhare of the money: I demand how much each man caught?

 Let

* This regiſter in the margin denotes, that the quantity $7x$ in the 4th ſtep is to be tranſpoſed, and that following it, ſignifies, that the 5th ſtep is divided by 10.

Let x, y, and z denote the money caught by *A*, *B*, and *C* refpectively, then will $\frac{3x}{4} + \frac{2y}{4} + \frac{z}{4}$ reprefent the money laid down, and $\frac{3x + 2y + z}{12}$ the equal quantity each received :

Hence	1	$\frac{x}{4} + \frac{3x + 2y + z}{12} = 220 = a$ per queft.
	2	$\frac{y}{2} + \frac{3x + 2y + z}{12} = 176 = b$
	3	$\frac{3z}{4} + \frac{3x + 2y + z}{12} = 154 = c$
Now the value of x in the 1ft equation is	4	$x = \frac{12a - 2y - z}{6}$
and from the 3d equation	5	$x = \frac{12c - 10y - z}{3}$
4th = 5th	6	$24c - 20y - 4z = 12a - 2y - z$
and per 6th	7	$y = \frac{24c - 19z - 12a}{2}$
Let the 3d be taken from the 2d and the difference is	8	$\frac{y}{2} - \frac{3z}{4} = b - c$
and the value of y in the 8th is	9	$y = \frac{4b - 4c + 3z}{2}$
make the 7th = 9th	10	$4b - 4c + 3z = 2c - 19z - 12a$
Hence in the 10th	11	$z = \frac{24c - 6a - 2b}{11} = 44$ what *C* caught
confequently	*A*	caught 396
	B	—— 110
	C	—— 44*

Queft. 7th. There is a certain number, confifting of 2 figures, and it is equal to 4 times the fum of its digits, and if you add 18 to the number, the digits will be inverted : I demand the number ?

Let

* This queftion is the 12th in the Gentleman's Diary for the year 1770.

Let the digit in the tens place be x, and that in the place of units y,

	1	$10x + y = 4x + 4y$ } per quest.
	2	$10y + x = 10x + + 18$ } per quest.
By the 1st	3	$x = \frac{y}{2}$
and by the 2d	4	$x = y - 2$
$3^d = 4^{th}$	5	$y - 2 = \frac{y}{2}$
$5^{th} + 2$	6	$y = \frac{y}{2} = 2$
$6th - \frac{y}{2}$	7	$\frac{y}{2} = 2$
$7th \times 2$	8	$y = 4$ the digit in the units place
and	9	$x = y - 2 = 2$ the digit in the place of tens.

Hence the required number is 24.

Proof, $\overline{2+4} \times 4 = 24$ and $24 + 18 = 42$ which inverts the said digits.

Quest. 8th. What number is that, the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of itself, is less than the said number by 1?

Let the required number be x,

Then will	1	$\frac{x}{3} + \frac{x}{4} + \frac{x}{5} + \frac{x}{6} = x - 1$
$1st \times 6$	2	$2x + \frac{3x}{2} + \frac{6x}{5} + x = 6x - 6$
2×5	3	$10x + \frac{15x}{2} + 6x + 5x = 30x - 30$
3×2	4	$20x + 15x + 12x + 10x = 60x - 60$
by adding the co-efficients x	5	$57x = 60x - 60$
$5 + 60$	6	$60 + 57x = 60x$
$6 - 57x$	7	$60 = 3x$
$7 \div 3$	8	$x = 20$ the answer.

For $\frac{x}{3} = 6, \dot{6}$

$\frac{x}{4} = 5$

$\frac{x}{5} = 4$

$\frac{x}{6} = 3, \dot{3}$

20 proof

Quest. 9th. I am a brazen lion, my two eyes, my mouth, and the sole of my right foot are so many several pipes, they fill a cistern, the right eye fills it in two days, the left in 3, and the sole of my foot in 4; but my mouth can fill it in 6 hours: tell me me in what time they can fill the cistern when they run all together.

Let	1	x the required time;
then $6 : 1 ::$	x	$: \frac{x}{6}$ the quantity carried into the cistern by the mouth,
$48 : 1 ::$	x	$: \frac{x}{48}$ the quantity carried by the right eye,
$72 : 1 ::$	x	$: \frac{x}{72}$ the quantity carried by the left eye,
$96 : 1 ::$	x	$\frac{x}{96}$ the quantity carried by the sole of right foot.
consequently	2	$\frac{x}{6} + \frac{x}{48} + \frac{x}{72} + \frac{x}{96} = 1$
2×96	3	$16x + 2x + \frac{4x}{3} + x = 96$
3×3	4	$48x + 6x + 4x + 3x = 288$
4 added	5	$61x = 288$
$5 \div 61$	6	$x = \frac{288}{61} = 4\frac{44}{61}$ the time required.

Quest.

Queſt. 10th. The Epicurean *Greeks*, accounted their thrium amongſt their moſt delicious dainties. It was a kind of cake or wafer of a determinate weight, $\frac{1}{3}$ of it was of the fineſt wheat flower, $\frac{1}{6}$ of it was of eggs together with an ounce and half of lard, and the ſame of honey, to theſe they added a hemina of milk, which cantained 9 ounces, theſe mixed in this proportion, were baked upon a fig leaf: I demand the weight of the whole cake?

	1	$x =$ the weight of the whole cake,
	2	$\frac{x}{3}$ the weight of the flower,
	3	$\frac{x}{6} =$ the eggs,
	4	3 ounces of lard and honey
	5	9 ounces of milk.
Weight of the whole	6	$\frac{x}{3} + \frac{x}{6} + 12 = x$
6×6	7	$2x + x + 72 = 6x$
$7 - 3x$	8	$72 = 3x$
$8 \div 3$	9	$x = 24$ ounces the weight of the whole cake.

ARITHMETICAL PROGRESSION.

157. HOW this kind of progreſſion is formed, has already been ſhewn, (art 48) we come now to treat of the properties of quantities thus progeſſional.

158. If three quantities are in arithmetical progreſſion, the ſum of the extreams is equal the double of the mean; thus let us take 2, 6, 10, and 2×10, is $= 6 \times 2 = 12$.

159. If four quantities are in arithmetical progreſſion, the ſum of the means is equal the ſum of the extreams; for in 2, 6, 10, 14, we have $6 + 10 = 16$ and $2 + 14 = 16$ alſo.

160. Again, if a series of numbers, are in arithmetical progression, the double of any term in the series, is equal the sum of any two terms equally remote from the said term.

EXAMPLE.

Let 1, 3, 5, 7, 9, 11, 13, 15, be the given series, and we have $9 \times 2 = 15 + 3 = 7 + 11 = 13 + 5$.

161. If we take an increasing and decreasing series of equal value, and the same number of terms, we shall have the sum of every term equal;

Thus $\begin{Bmatrix} 1, & 3, & 5, & 7, & 9, & 11, & 13, & 15, & 17 \\ 17, & 15, & 13, & 11, & 9, & 7, & 5, & 3, & 1 \end{Bmatrix}$

Sum 18, 18, 18, 18, 18, 18, 18, 18, 18

162. Now, in the foregoing example, there are 9 terms, and the sum of every term alike, viz. = to 18, therefore $\frac{9 \times 18}{2}$ must express the sum of one of these series, and hence we derive the following general rule:

RULE. *Multiply the sum of the greatest and least terms by half the number of terms, and the product is the sum of the whole progression.*

163, If we put $g =$ the greatest term, $l =$ the least, $n =$ the number of terms, $d =$ the common difference, and $s =$ the sum of the whole progression, we will per rule, get $\overline{g + l} \times \frac{n}{2} = S$ theorem 1st,

$\frac{2S}{g+l} = n$ theorem 2d,

$\frac{2s - gn}{n} = l$ theorem 3d,

$\frac{2s - ln}{n} = g$ theorem 4th

164. But ſeeing the number of terms muſt neceſſarily be one more than the number of common differences, *if*, therefore, *the difference between the greateſt and leaſt terms, be divided by the number of terms, minus one, the quotient will be the common difference*, that is in ſymbols $\frac{g-l}{n-1}=d$ theorum 5th

$$\overline{n-1}\times d+g=l \text{ theorem 6th,}$$
$$\overline{n-1}\times d+l=g \text{ theorem 7th,}$$
$$\frac{g-l}{d}+1=n \text{ theorem 8th.}$$

165. I need ſcarce recommend it to the young algebraiſt, to make uſe of theſe theorems, rather than rules written at full length, for it will eaſily occur to him, that the ſymbols are all under his eye at one view, whereas, a long and tedious rule requires time and conſideration in reading every member thereof: yet, for the benefit of ſuch as do not underſtand algebra, I ſhall put the foregoing eight theorems into words.

Theorem 1. *The ſum of the greateſt and leaſt terms, multiplied by half the number of terms, is equal the ſum of the whole progreſſion.*

Theorem 2d. *Twice the ſum of the whole progreſſion, divided by the ſum of the greateſt and leaſt terms, is equal to the number of terms.*

Theorem 3d. *The double ſum of the progreſſion, divided by the number of terms, and the greateſt term taken from the quotient, is equal to the leaſt term.*

Theorem 4th. *The double ſum of the whole pregreſſion, divided by the number of terms, and the leaſt term taken from the quotient, is equal the greateſt term.*

Theorem 5th. *The difference between the greateſt and leaſt terms divided by one leſs than the number of terms, is equal the common difference.*

Theorem

Theorem 6th. *The number of terms, minus one, multiplied by the common difference, and the product added to the greatest term, is equal to the least.*

Theorem 7th. *The number of terms, minus one, (or unity) multiplied by the common difference, and the product added to the least term, is equal the greatest.*

Theorem 8th. *The difference between the greatest and least terms, divided by the common difference, and one added to the quotient, gives the number of terms.*

EXAMPLES.

Queſt. 1ſt. If a hundred eggs are placed in a right line, at a yard diſtance from each other, and the firſt a yard from a baſket; required how far a perſon muſt travel who gathers them into the baſkit one by one?

Here are given $\left\{\begin{array}{lcl} \text{the leaſt term} & = & 2 = l \\ \text{the greateſt ditto} & = & 200 = g \\ \text{the number of terms} & = & 100 = n \end{array}\right\}$ to find the ſum of the progreſſion $= s$, which, per theorem 1ſt, is $\overline{200 + 2} \times \frac{100}{2} = 101000$ yards, which is $5\frac{3}{4}$ miles nearly.

Queſt 2d. A debtor has agreed to pay a debt of 18*l* 10*s* 6*d* at weekly payments; he is to pay down 5 ſhillings, and make an equal encreaſe of payment at every weeks end following, ſo that the laſt payment ſhall be 9*s*. 3*d*. required the number of weeks to pay off the debt, and the difference of each weekly payment?

We have here given $\left\{\begin{array}{lcccl} \text{the ſum of prog.} = s = & 18l.\ 10s.\ 6d. & = & 4446d. \\ \text{the leaſt term} = l = & 5 & = & 60 \\ \text{the greateſt term} = g = & 9\ \ 3 & = & 111 \end{array}\right\}$ to find $n =$ the number of terms, and $d =$ the common difference; and per theorem 2d, $\frac{4446 \times 2}{60 + 111} = 52$ weeks the number of terms; and again, per theorem 5th $\frac{111 - 60}{52 - 1} = 1$ penny the weekly or common difference.

Geome-

GEOMETRICAL PROGRESSION.

166. HOW quantities in geometrical progreſſion are generated, was defined in art. 49th. but if three numbers are in geometrical progreſſion, the product of the extreams is equal the ſquare of the mean.

Example. 3, 9, 27, for $3 \times 27 = 9 \times 9 = 81$.

167. Four quantities in geometrical progreſſion, the product of the extreams is equal the product of the means.

Example. 3, 9, 27, 81, where $3 \times 81 = 27 \times 9 = 243$.

168. If a ſeries of numbers in arithmetical progreſſion, beginning with a cypher, and the common difference, equal to unity, or one be placed over a ſeries of numbers in geometrical progreſſion, proceeding from unity; thus $\left\{\begin{matrix} 0. & 1. & 2. & 3. & 4. & 5. & 6. & 7. & 8. & 9. \\ 1. & 2. & 4. & 8. & 16. & 32. & 64. & 128. & 256. & 512. \end{matrix}\right\}$ we may obſerve the following particulars, viz,

169. That the ſum of any two or more terms in the arithmetical ſeries correſponds with the products of the terms below them in the geometrical ſeries.

Example. $\left\{\begin{matrix} 2 + 3 + 4 = 9 & \text{in the Arith. ſeries,} \\ 4 \times 8 \times 16 = 512 & \text{in the Geo. ſeries.} \end{matrix}\right.$

170. That if the difference between any two terms in the arithmetical ſeries be taken, it will correſpond with the diviſion of the terms in the geometrical ſeries.

Example $\left\{\begin{matrix} 7 - 3 = 4 & \text{in the arith. ſeries.} \\ 128 \div 8 = 16 & \text{in the geo. ſeries.} \end{matrix}\right.$

171. That any term in the arithmetical ſeries multiplied by 2, 3, 4, &c. agrees with an involution of the correſponding term to the 2d, 3d, 4th, &c. power.

Example $\begin{cases} 3 \times 2 = 6 \text{ in the arithmetical ſeries.} \\ 8 \times 8 = 64 \text{ in the geometrical ſeries.} \end{cases}$

172. That any term in the arithmetical ſeries, divided by 2, 3, 4, &c. will anſwer to an evolution of the correſponding term in the geometrical ſeries.

Example $\begin{cases} 4 \div 2 = 2 \text{ in the arith. ſeries.} \\ \sqrt{16} = 4 \text{ in the Geo. ſeries.} \end{cases}$

173. Let us now re-aſſume the ſeries delivered at art. 123, viz. $\frac{ee}{e-a} = e + a + \frac{a^2}{e} + \frac{a^3}{e^2} + \frac{a^4}{e^3}$ &c. where we obſerve the terms decreaſe in geometrical progreſſion, the firſt or greateſt term being e, the ſecond a, and the ratio $\frac{a}{e}$. And ſeeing this ſeries is generated from $\frac{ee}{e-a}$, and the ſum of all the terms thence ariſing *ad infinitum* can never exceed the quantity they are generated from; for the ſum of the parts of any body or quantity cannot exceed the whole: and from hence the following general rule is formed.

RULE I.

Divide the ſquare of the firſt or greateſt term by the difference between the two greateſt terms, and the quotient is the ſum of the infinite progreſſion in finite terms.

174. Again, let us take the next example in the ſaid article, viz. art. 123, where $\frac{ex}{e-1} = x + \frac{x}{e} + \frac{x}{e^2} + \frac{x}{e^3}$ &c. and it is eaſy to obſerve, that the firſt term is x

and

and the ratio $\frac{1}{e}$, or that the ſeries is decreaſing in geometrical progreſſion, and hence we have another general rule for finding the ſum of an infinite geometrical ſeries, viz.

RULE 2d. *Multiply the greateſt term by the common diviſor or ratio, and divide the product by one leſs than the ſaid ratio, the quotient is the ſum of the infinite progreſſion.*

175. Put $g =$ the greateſt term and $r =$ the ratio, and the following ſeries will be expreſſed, viz. $g + gr + gr^2 + gr^3 + gr^4$ &c. now, if S be put for the ſum of the infinite ſeries, we will get,

per rule 1ſt. $\frac{g^2}{g - gr} = S$ theorem 1ſt,

and, per rule 2d, $\frac{gr}{r - 1} = S$ theorem 2d.

176. Now where the terms are infinite, the leaſt term will be infinitely little or equal to nothing; but if the terms are finite, the leaſt term will be ſome aſſignable quantity, which put equal l, then it is very evident that $\frac{gr}{r-1} - \frac{l}{r-1}$ muſt be the ſum of the finite ſeries, viz. $\frac{gr - l}{r - 1} = S$ theorem 3d: hence the following rule.

RULE 3d. *Multiply the greateſt term by the ratio, and from the product take the leaſt term, the difference divided by one leſs than the ratio is the ſum of the whole finite progreſſion.*

177. In the progreſſion $l + lr + lr^2 + lr^3 + lr^4$ &c. it appears from inſpection, that the number of terms $= n$, is one more than the index of the ratio; hence lr^{n-1} will be a general expreſſion for the greateſt

 term

term: whence, per rule 3d, we get $\frac{lr^{n-1} \times r : - l}{r - 1}$

that is $\frac{lr^n - l}{r - 1} = S$ theorem 4th, which in words affords the following general rule.

RULE 4. *Multiply the least term by the ratio involved to a height equal the number of terms, from which take the least term, the difference divided by one less than the ratio, is the sum of the whole progression.*

178. By finding the value of r in the 3d theorem, we have the following, viz. $\frac{s - l}{s - g} = r$ theorem 5th, and in like manner may the value of any letter in the third and fourth theorems be found; but as the theorems arising from thence are not in so much use as those we have given, and being easily found from the foregoing principles, we shall not insert them or the rules, but proceed to

EXAMPLES.

Quest. 1st. Suppose a bullet fly 20 miles the first second of time, 19 miles the second, $18\frac{1}{20}$ the 3d, &c. in the same geometrical decreasing progression, how far will it fly in a whole eternity?*

Per rule 1st, $\frac{20 \times 20}{1} = 400$ miles the answer;

or, per rule 2d, $\frac{20 \times \frac{20}{19}}{\frac{1}{19}} = 400$ miles also.

Quest. 2d. Required the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, &c. *ad infinitum?*

Per rule 1st, $\frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{4}} = 1$ the answer;

or, per rule 2d, $\frac{\frac{1}{2} \times 2}{1} = 1$ also.

Quest

* This question is n Mr Martin's Miscellany.

Queſt. 3d. A ſervant agrees with a gentleman to ſerve him 2 years or 24 months, on the following conditions, viz. to have a farthing the firſt month, 2 the ſecond, 4 the third, 8 the fourth, &c. doubling every payment to the end of the 24 months. Quere his wages?

Let a few of the leading terms ſtand as under.

$$\left.\begin{array}{lccccccc} \text{Arithmatical ſeries} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{Geometrical ſeries} & 1 & 2 & 4 & 8 & 16 & 32 & 64 \end{array}\right\} \text{then}$$

(by art. 172) find the laſt term, and becauſe $6 + 6 = 12$, therefore $64 \times 64 = 4096$ the twelfth term, which multiplied by 32, viz. the 5th term gives the 17th, &c. See the work.

$$\begin{array}{rl}
64 & \text{the 6th term,} \\
64 & \\
\hline
256 & \\
384 & \\
\hline
4096 & \text{the 12th term,} \\
32 & \text{the 5th term,} \\
\hline
8192 & \\
12288 & \\
\hline
131072 & \text{the 17th term,} \\
64 & \text{the 6th term,} \\
\hline
524288 & \\
786432 & \\
\hline
8388608 & \text{the 23d term,}
\end{array}$$

and becauſe the index of the firſt term is a cypher, therefore this term is the laſt or twenty-fourth payment, which the learner muſt be careful to obſerve in ſimilar caſes, then per rule 3d, $\overline{8388608 \times 2} - 1 = 16777215$ farthings reduced to pounds is 17476*l.* 5*s.* $3\frac{3}{4}$*d.* the anſwer.

Queſt. 4th. Doctor Wallace informs us, that one Seſſa, an Indian, was the firſt who diſcovered the amazing power of numbers in geometrical progreſſion, having invented the game at cheſs and ſhewn it to his Prince Shebram, who was ſo highly pleaſed with the game, that he bid him aſk what ever he pleaſed as a reward for his ingenuity; whereupon he requeſted one grain of wheat for the firſt ſquare in the cheſs board, two for the ſecond, &c. doubling continually for every ſquare in the whole board, which were 64. The King was highly diſpleaſed at his aſking ſo trifling a reward; but much more ſurprized when he found his whole empire could not ſupply the demand. Quere the number of grains and (allowing, according to the ſtandard, 7860 grains to a pint) how many buſhels will it amount to?

Now, per art. 172, the laſt term will be found to come out 9223372036854775808 and, per rule 3d, the ſum of the progreſſion is 18446744073709551615 grains, which, divided by 503040, the grains in a buſhel will amount to upwards of 30000000000000 buſhels, which, at the moderate price of 4 ſhillings per buſhel, will amount to upwards of one billion or a million millions of pounds ſterling.

179. We ſhall now propoſe a few examples in the practical part of algebra in order to illuſtrate the ſolution

Of QUADRATIC EQUATIONS.

Queſt. 11 THERE are three numbers in geometrical progreſſion, their ſum is 74, and the ſum of their ſquares 1924; required the numbers?

Let

Let the three numbers be repreſented by x, y, z,

Then	1	$x + y + z = 74 = s$ per. queſt.
	2	$x^2 + y^2 + z^2 = 1924 = a$ per. queſt.
	3	$xz = y^2$ per art. 169
3d × 2	4	$2xz = 2y^2$
$1 - y$	5	$x + z = s - y$
$2 - y^2$	6	$x^2 + z^2 = a - y^2$
4 and 6 added	7	$x^2 + 2xz + z^2 = a + y^2$
5 ſquared	8	$x^2 + 2xz + z^2 = s^2 - 2sy + y^2$
7 put = 8	9	$s^2 - 2sy + y^2 = a + y^2$
$9 - y^2$	10	$s^2 - 2sy = a$
$10 + 2sy$	11	$s^2 = 2sy + a$
$11 - a$	12	$s^2 - a = 2sy$
$12 \div 2s$	13	$y = \dfrac{s^2 - a}{2s} = 24$
4 × 4	14	$4xz = 4y^2$
14 taken from the 7	15	$x^2 - 2xz + z^2 = a - 3y^2$
15 ſquare root extracted	16	$x - z = \sqrt{a - 3y^2}$
5 added to the 16	17	$2x = \sqrt{a - 3y^2} + s - y$
$17 \div 2$	18	$x = \dfrac{\sqrt{a - 3y^2} + s - y}{2} = 32$
16 tranſpoſed	19	$z = x - \sqrt{a - 3y^2} = 18$.

Queſt. 12th. What two numbers are thoſe, the ſum of which is 20 and product 36?

Put x for the greater and y for the less number,

Then	1	$x + y = 20 = s$ } per question
	2	$xy = 36 = p$
$1 - y$	3	$x = s - y$
$2 \div y$	4	$x = \frac{p}{y}$
$3^{d} = 4^{th}$	5	$s - y = \frac{p}{g}$
$5 \times y$	6	$sy - yy = p$
6 transposed	7	$y^2 - sy = - p$*
7 made a square N° by adding $\frac{s^2}{4}$	8	$y^2 - sy + \frac{s^2}{4} = \frac{s^2}{4} - p$
8 ɯ 2, or the square root taken	9	$y - \frac{s}{2} = \sqrt{\frac{s^2}{4} - p}$
$9 + \frac{s}{2}$	10	$y = \frac{s}{2} \pm \sqrt{\frac{s^2}{4} - p} = 2$
per 3d	11	$x = s - y = 18$

180. When a Quadratic Equation is produced it will appear in one of the following forms.

viz.	$x^2 + sx = p$	the square being compleated of which c □ is the sign we get.	$x^2 + sx + \frac{s^2}{4} = p + \frac{s^2}{4}$
	$x^2 - sx = p$		$x^2 - sx + \frac{s^2}{4} = p + \frac{s^2}{4}$
	$x^2 - sx = - p$		$x^2 - sx + \frac{s^2}{4} = \frac{s^2}{4} - p$

The

* The unknown quantity in this step is both to the second and first power, and the methods of solutions hitherto delivered would prove ineffectual in clearing the equation; if we therefore consult Art. 140, we shall find that if $\frac{s^2}{4}$ is added to both sides of the equation as is done at the eight step, the quantity $y^2 - sy + \frac{s^2}{4}$ being one of the sides of the equation is a complete square number, and of consequence its root may be had agreeable to the following step.

The Square Root being extracted they become as below.

$$\left.\begin{array}{l} x+\frac{s}{2}=\sqrt{p+\frac{s^2}{4}} \\ x+\frac{s}{2}=\sqrt{p+\frac{s^2}{4}} \\ x+\frac{s}{2}=\sqrt{\frac{s^2}{4}-p} \end{array}\right\} \text{by transposition} \left\{\begin{array}{l} x=\sqrt{p+\frac{s^2}{4}}:-\frac{s}{2} \\ x=\sqrt{p+\frac{s^2}{4}}:-\frac{s}{2} \\ x=\frac{s}{2}\pm\sqrt{\frac{s^2}{4}-p}\,^{*} \end{array}\right.$$

181. From the last article we deduce the following general and practical Rule for compleating the Square.

RULE. *Take the square of half the co-efficient of the unknown quantity (at the first power) which add to both sides of the equation, by which means that side of the equation which contains the unknown quantity is made a complete square.*

Quest. 13. *A* has a joint note from *B* and *C* for 174 *l.* they being unable to pay when demanded, agreed that *B* should pay 8 *l.* every month and *C* 1 *l.* the first month, 2 the second, 3 the third, 4 the fourth, &c. till the debt was discharged. Query the time and what each paid?

Let

* Every quadratic equation has two roots, one of which, in the two former equations, is always negative or imaginary; but in the latter they are both afirmative: and hence, to the last step is put the double sign $\pm$, which signifies, that the quantity after it may be added or subtracted according as the conditions of the question require. $x-\frac{s}{2}$ and $\frac{s}{2}-x$ squared, give the same quantity which is the reason of a quadratic having two roots.

Let	1	x the time or number of payments
	2	then $8x$ the money paid by B
and per progreſſion	3	$\overline{1+x} \times \frac{x}{2} = \frac{x+xx}{2}$ ditto paid by C
2d + 3d	4	$\frac{x+xx}{2} + 8x = 174$ per queſtion
4×2	5	$x + xx + 16x = 348$
5 ∴	6	$xx + 17x = 348$
6 c □	7	$xx + 17x\ 72{\cdot}25 = 420{\cdot}25$
7 ш 2	8	$x + 8{\cdot}5 = 20{\cdot}5$
$8 - 8{\cdot}5$	9	$x = 12$ months the time
per 2d	10	$8x = 96\,l.$ what B paid
per 3	11	$\frac{x+xx}{2} = 78\,l.$ what C paid.

Queſt. 14th. There is a certain number conſiſting of two digits, and it is equal to four times the ſum of thoſe digits; but if 18 be added to the number, the ſaid digits will be inverted. Query the numbers?

Let the digit in the units place be x and that in the place of tens y:

Then	1	$10y + x = 4y + 4x$ } per queſt.
and	2	$10y + x + 18 = 10x + y$ } per queſt.
1ſt tranſpoſed	3	$6y = 3x$
$3 \div 3$	4	$2y = x$
2d $- y$	5	$9y + x + 18 = 10x$
$5 - x$	6	$9y + 18 = 9x$
4×9	7	$18y = 9x$
7th = 6th	8	$18y = 9y + 18$
$8 - 9y$	9	$9y = 18$
$9 \div 9$	10	$y = 2 =$ the digit in the ten's place
per 4th	11	$x = 2y = 4$ the digit in the unit's place
hence	12	$10y + x = 24$ the required number

For $\overline{4+6} \times 4 = 24$ and $24 + 18 = 42$.

Queſt. 15th. Given the product of two numbers $= p$, and the ſum of their ſquares $= s$, required the numbers?

Let

Let the greater be x and the less y:

Then	1	$xy = p$ } per question.
	2	$xx + yy = s$
$1 \div y$	3	$x = \frac{p}{y}$
For xx put its equal $\frac{pp}{yy}$	4	$\frac{pp}{yy} + yy = s$
$4 \times yy$	5	$pp + y^4 = sp^2$
$5 - pp$	6	$y^4 = sp^2 - pp$
$6 - sp^2$	7	$y^4 - sp^2 = -pp$
7 c □	8	$y^4 - sp^2 + \frac{s^2}{4} = \frac{s^2}{4} - pp$
8 ш 2	9	$y^2 - \frac{s}{2} = \sqrt{\frac{s^2}{4} - pp}$
$9 + \frac{s}{4}$	10	$y^2 = \frac{s}{2} \pm \sqrt{\frac{s^2}{4} - pp}$
10 ш 2	11	$y = \sqrt{\frac{s}{2} \pm \sqrt{\frac{s^2}{2} - pp}}$
per 3d	12	$x = \frac{p}{y}$

Of SIMPLE INTEREST.

Put {
p = the principal or sum lent
r = the ratio of the rate per cent. *
t = the time of continuance.
a = the amount of the principal and interest.

* The ratio of the rate is the interest of 1 *l.* for one year, at any given rate per cent.

A TABLE *of the usual* RATIOS *of the* RATES *per cent.*

l.		*l.*		*l.*		*l.*			*l.*	*s.*	
100	:	5	: :	1	:	·05	= *r*	at	5	0	per cent.
100	:	4·5	: :	1	:	·045	= *r*	at	4	10	per cent.
100	:	4	: :	1	:	·04	= *r*	at	4	0	per cent.
100	:	3·5	: :	1	:	·035	= *r*	at	3	10	per cent.
100	:	3	: :	1	:	·03	= *r*	at	3	0	per cent.
100	:	2·5	: :	1	:	·025	= *r*	at	2	10	per cent.

If r be = the intereſt of 1 *l.* for 1 one year

Then $\begin{cases} 2\,r = \text{the intereſt of 1 } l. \text{ for 2 years} \\ 3\,r = \text{the intereſt of 1 } l. \text{ for 3 years} \\ \text{and } tr = \text{the intereſt of 1 } l. \text{ for } t \text{ years.} \end{cases}$

But 1 *l.* : *tr l.* : : *p l.* : *prt* the intereſt of p pounds in t years; and becauſe the principal added to the intereſt, gives the amount: therefore

$prt + p = a$ theorem 1ſt,

and the value of the principal from the ſaid equation is found $\Big\}$ $\dfrac{a}{tr+1} = p$ theorem 2d,

alſo the ratio is $\dfrac{a-p}{tp} = r$ theorem 3d,

and the time is $\dfrac{a-p}{rp} = t$ theorem 4th.

For the ſake of ſuch as do not underſtand Algebra, we ſhall give the foregoing theorems in words.

Theorem 1ſt. *The principal time and ratio multiplied together, and the product added to the principal, gives the amount.*

Theorem 2d. *Divide the amount by the product of the ratio plus, or more, one which is equal the principal.*

Theorem

Theorem 3d. *The difference between the amount and principal, divided by the product of the time and principal, is equal the ratio of the rate per cent.*

Theorem 4th. *The difference between the amount and principal, divided by the product of the ratio of the rate per cent. and principal, gives the time.*

EXAMPLES.

183. Queſt. 1ſt. Required the amount of 765 *l.* 10 *s.* at 4 *l.* per cent. per annum, for $7\frac{1}{2}$ years.

$$\text{Given} \left\{ \begin{array}{l} p = 765\cdot5 \\ r = \quad\ \cdot04 \\ t = \quad 7\cdot5 \end{array} \right\} \text{to find } a$$

Per theorem 1ſt. $\overline{765\cdot5 \times \cdot04 \times 75} + 765\cdot5 = 995\cdot15\ l.$ $= 995\ l.\ 3\ s.\ 0\ d.$ the anſwer.

Queſt. 2d. What ſum of money put to intereſt at 4 *l.* per cent per annum, will amount to 995 *l.* 3 *s.* in $7\frac{1}{2}$ years?

$$\text{Given} \left\{ \begin{array}{l} a = 995\cdot15 \\ r = \quad\quad \cdot04 \\ t = \quad 7\cdot5 \end{array} \right\} \text{to find } p \text{ per}$$

RULE 2d. $\dfrac{995\cdot15}{7\cdot5 \times \cdot04 + 1} = 765\cdot5\,l. = 765\,l.\ 10\,s.$ the anſwer.

Queſt. 3d. At what rate per cent per annum will 765 *l.* 10 *s.* amount to 995 *l.* 3 *s.* in $7\frac{1}{2}$ years?

$$\text{Given} \left\{ \begin{array}{l} a = 995\cdot15 \\ p = 756\cdot5 \\ t = \quad 7\cdot5 \end{array} \right\} \text{to find } r.$$

per Theorem 2d. $\dfrac{995\cdot15 - 765\cdot5}{7\cdot5 \times 765\cdot5} = 04 = r,$ which is $= 4\,l.$ per cent.

Queſt.

Queſt. 4th. In what time will 765 *l.* 10 *s.* amount to 995 *l.* 3 *s.* at 4 *l.* per cent per annum?

$$\text{Given}\left\{\begin{array}{l} a = 995{\cdot}15 \\ p = 765{\cdot}5 \\ r = \quad {\cdot}04 \end{array}\right\}\text{to find } t.$$

$$\text{Per Theorem 4th. } \frac{995{\cdot}15 - 765{\cdot}5}{{\cdot}04 \times 765.5} = t = 7{\cdot}5 \text{ years.}$$

Of ANNUITIES, PENSIONS, *&c.* *in arrear at* SIMPLE INTEREST.

DEFINITION.

184. ANNUITIES are in arrear when the yearly rent or other ſtated payments are unpaid for any given time.

Let u repreſent any annuity or ſtated payment,
r the ratio of the rate per cent.
t the time or numer of payments,
a the amount of the annuity and intereſt.

It it is plain that the annuity or ſtated payments, without any intereſt due at the end of t payments, will be tu, and the intereſt at the end of the ſecond payment will be ru,* at the end of 3d payment $2ru$, at the end of the fourth payment $3ru$, and conſequently at the end of t payments $\overline{t-1} \times ru$: and, therefore, the intereſt will be expeſſed by the following arithmetical ſeries, viz.

ru

* For $1 : r :: u : ru$.

$ru + 2ru + 3ru + 4ru$ &c. $\overline{t-1} \times ru$ being the laſt term of the ſeries: and hence, per rule, for the ſum of the arithmetical ſeries, we get $\overline{ru + \overline{t-1} \times ru} \times \frac{\overline{t-1}}{2} =$

$$\frac{ttr - tr}{2} + t \times u = a \text{ theorem 1ſt,}$$

$$\frac{2a}{ttr - tr + 2t} = u \text{ theorem 2d,}$$

$$\frac{2a - 2tu}{\overline{tt - t} \times u} = r \text{ theorem 3d,}$$

$$\sqrt{\frac{2a}{ru} + \frac{\overline{\frac{2}{r} - 1}|^2}{4}} - \frac{\frac{2}{r} - 1}{2} = t \text{ theorem 4th.}$$

Annuities, &c. are ſo ſeldom computed at ſimple intereſt, that I chuſe to omit the inſertion of any examples, nevertheleſs, thoſe who pleaſe to exerciſe themſelves therein, may eaſily propoſe examples of their own, and perform them agreeable to the above theorems.

Of COMPOUND INTEREST.

185. LET (as in ſimple intereſt) $p =$ the principal $r =$ the ratio of the rate per cent. per annum,* $t =$ the time, and $a =$ the amount.

* The amount of 1 pound in 1 year is termed the ratio.

A TABLE *of* RATIOS.

l.		*l.*		*l.*		*l.*		*l.*	*s.*	
100	:	105	: :	1	:	1·05	$= r$ at	5		per cent.
100	:	104·5	: :	1	:	1·04·5	$= r$ at	4	10	per cent.
100	:	104	: :	1	:	1·04	$= r$ at	4		per cent.
100	:	103·5	: :	1	:	1·04 5	$= r$ at	3	10	per cent·
100	:	103	: :	1	:	1·03	$= r$ at	3		per cent.
100	:	102·5	: :	1	:	1·02·5	$= r$ at	2	10	per cent.

Now as $1 l. : r l. :: r l. : r^2 l.$ the amount of 1*l.* at the end of 2 years, and $1 : r :: r^2 : r^3$ the amount of 1*l.* at the end of 3 years; but generally as $1 l. r :: r^{t-1} : r^t$ the amount of 1*l.* in *t* years, consequently as $1 l. : r^t l. :: p l. : p r^t l.$ therefore $p r^t = a$ a general theorem, by which the others are easily discovered; but, as in each of them, the index of *r* is *t*, the calculation becomes very tedious, especially when the time is long, in order therefore to facilitate the work, we must have recourse to logarithms, the properties of which have been illustrated in the geometrical series, art. 170. &c, and seeing there are tables in almost every Arithmetician's hands, or at least they may be purchased by themselves as cheap as we can afford to print them, 'tis judged unnecessary to give them a place in this book.

186. And since $p r^t = a$, we have from the nature of the logerithms $l.r \times t + l.p = l \cdot a$* theorem 1st,

$$l.a - l.r \times t = l.p \text{ theorem 2d,}$$

$$\frac{l.a - l.p}{t} = l.r \text{ theorem 3d,}$$

$$\frac{l.a - l.p}{l.r} = t \text{ theorem 4th.}$$

187.

* When *l* is prefixed to any quantity, it denotes the logarithm of that quantity.

187. These theorems will give the following words;

Theorem 1st. Multiply the logarithm of the ratio, by the time, to which add the logarithm of the principal, which sum is equal the logarithm of the amount.

Theorem 2d. From the log. of the amount, take the log. of the ratio, multiplied by the time, the difference is equal the logarithm of the principal.

Theorem 3d. The Logarithm of the principal taken from the logarithm of the amount, and the difference divided by the time, gives the logarithm of the ratio.

Theorem 4th. The logarithm of the principle, taken from the logarithm of the amount, and the difference divided by the logorithm of the ratio, gives the time.

EXAMPLES.

Quest. 1st. Required the amount of 765*l.* 10*s.* at 4*l.* per cent per annum in $7\frac{1}{2}$ years, at compound interest.

Per theorem 1st. The logorithm of the ratio $= 1\cdot04$ is	0·0170333
multiplied by the time	7·5
	0851665
	1192331
the product is the logarithim of $r^t =$	0·12774975
to which add the logarithm of $p = 765\cdot5 =$	2·88394520
the sum is logorithm of the amount $= a = 1027\cdot294 =$	3·01169459

and hence the amount at compound interest is more than the amount at simple interest by 31*l.* 10*s.* $10\frac{1}{2}$*d.* as appears from a comparison of the examples.

Quest. 2d. What sum of money at 4*l.* per cent per annum, will amount to 1027*l.* 5*s.* $10\frac{1}{2}$*d.* in $7\frac{1}{2}$ years compound interest?

Per theorem 2d. From the log. of the amount $= a = 1027{\cdot}294\,l. =$ } $3{\cdot}0116949$

take the log. of $r = 1{\cdot}04 = {\cdot}017033 \times 7{\cdot}5 = 0{\cdot}1277497$

the difference is the log. of $p = 765{\cdot}5 = 2{\cdot}8839452$

Queſt. 3d. At what rate per cent. per annum, compound intereſt, will 765*l.* 10*s.* amount to 1027*l.* 5*s.* $10\frac{1}{2}$*d.* in $7\frac{1}{2}$ years?

Per theorem 3d. From the log. of $a = 1027{\cdot}294\,l. =$ } $3{\cdot}0116949$

take the log. of $p = 765{\cdot}5 = 2{\cdot}08839452$

$0{\cdot}1277497$

then $\frac{0{\cdot}1277497}{7{\cdot}5} = {\cdot}0170333 =$ the log. of the ratio $r = 1{\cdot}04 = 4\,l.$ yer cent.

Queſt. 4th. In what time will 765*l.* 10*s.* amount to 1027*l.* 5*s.* $10\frac{1}{2}$*d.* at 4*l.* per cent. per annum, compound intereſt?

Per theorem 4th. From the log. of $a = 1027{\cdot}294\,l. =$ } $3{\cdot}01169495$

take the log. of $p = 765{\cdot}5\,l. = 2{\cdot}8839452$

$0{\cdot}12774975$

hence $\frac{0{\cdot}12774975}{{\cdot}070333} = 7{\cdot}5 = t$ the time.

Of ANNUITIES, *&c. at* COMPOUND INTEREST.

188. HERE alſo let $u =$ the annuity, r the ratio or amount of 1*l.* and its intereſt, t the time, and

and a = the amount: then as $1\,l.\ :\ r\,l.\ ::\ u\,l.\ r\,u\,l.$ the interest and annuity due at the end of the second year, and $1\,l.\ :\ r\,l\ ::\ r\,u\,l.\ :\ u\,r^2$ the interest and annuity due for the third year, &c. hence we get

$$\left.\begin{array}{llllll} \text{years} & 1 & 2 & 3 & 4 & 5 \\ \text{amount due} & u & ru & ur^2 & ur^3 & ur^4 \end{array}\right\}$$ &c. where ur^{t-1} will be the last term of the series, and seeing the terms arising from the above series are in geomerical progression, we have the sum of the said series $\dfrac{ur^t - u}{r - 1} = a$: and hence we get the following logarithmical theorems;

$$l.u + l.\overline{r^t - 1} - l.\overline{r - 1} = l.a \text{ theorem 1st,}$$

$$l.a - l.\overline{r^t - 1} + l.\overline{r - 1} = l.u \text{ theorem 2d,}$$

$$\frac{l.\overline{ar - a + u} - l.u}{l.r} = t \text{ theorem 3d.}$$

When the time t is large, the fourth theorem for finding the ratio of the rate per cent. runs to a very high adfected equation, and becomes very tedious in calculation; we therefore omit it as being a matter of much labour and little use.

189. Theorem 1st. The logarithm of the annuity added to the logarithm of the ratio, involved to the power of the time, minus unity, and the logarithm of the ratio, minus unity, taken from the sum, gives the logarith of the amount.

Theorem 2d. The logorithm of the ratio, involved to the power of the time, minus unity, taken from the logarithm of the amount, and the logarithm of the ratio, minus unity, added to the difference, gives the logarithm of the annuity.

Theorem 3d. The logarithm of the annuity, taken from the logarithm of the amount, multiplied by the ratio, and the amount taken from the product, and the annuity added thereto; the difference divided by the logarithms of the ratio gives the time.

EXAMPLES.

Quest. 1st. If 50*l.* yearly rent or annuity, be foreborn or unpaid 7 years, what will it amount to at 4 *l.* per cent per annum, compound interest?

Multiply the logarithm of $r = 1\cdot04$	=	$\cdot0170333$
	by	$t = 7$
the product is $r^t = 1\cdot316$	=	$\cdot1192331$
then to the logarithm or $r^t - 1 = \cdot316$	=	$\bar{1}\cdot4996871$
add the logarithm of $u = 50\cdot$	=	$1\cdot6989700$
		$1\cdot1986571$
from which take the log. of $r - 1 = \cdot04$	=	$\bar{2}\cdot6020600$
the difference is the amount $= a = \cdot395$	=	$2\cdot5965971$

Quest. 2d. If 50*l.* per annum, payable quarterly, be foreborn 7 years, what will it amount to at 4*l.* per cent per annum, compound interest?

Here the annuity $= u = \frac{50}{4} = 12\cdot5$, the ratio $= r = 1\cdot01$, the time $= t = 28$, and therefore

Multiply the log. of $r = 1\cdot01$	=	$\cdot0043214$
	by	$t = 28$
		345712
		86428
the product is the log. of $r^t = 1\cdot322$	=	$\cdot1209992$
to the log. of $r^t - 1 = \cdot322$	=	$\bar{1}\cdot5078559$
add the log. of $u = 12\cdot5$	=	$1\cdot0969100$
the difference is the log. of $a = 402\cdot5$	=	$2\cdot6047659$

therefore we find the quarterly payments of more advantage than the yearly ones.

Quest

Queſt. 3d. What annuity, to continue 7 years, at 4*l*. per cent per annum, will amount to 395*l*. at compound intereſt?

$$\begin{array}{rr} \text{From the log. of } a = 395 = & 2\cdot5965971 \\ \text{take the log. of } r^t - 1 = \cdot316 = & 1\cdot4996871 \\ \hline & 3\cdot0969200 \\ \text{to which add the log. of } r - 1 = \cdot04 = & \bar{2}\cdot6020600 \\ \hline \text{the sum is the log. of } u = 50 = & 1\cdot6989700 \\ \hline \end{array}$$

Queſt. 4th. In what time will an annuity of 50*l*. per annum, amount to 395*l*. at 4*l*. per cent. per annum, compound intereſt?

$$\begin{array}{rr} \text{From the log. of } ar - a + u = 65\cdot8 = & 1\cdot8182259 \\ \text{take the log. of } u = 50 = & 1\cdot6989700 \\ \hline & \cdot1192559 \end{array}$$

Then $\frac{1192559}{0170333} = t = t$ = the time required.

Of the present WORTH *of* ANNUITIES.

190. THE method of inveſtigating the preſent worth of annuities is deduced from the two general theorems, viz. $pr^t = a$ and $\frac{ur^t - u}{r-1} = a$; and hence $pr^t = \frac{ur^t - u}{r-1}$; and conſequently, per the nature of the log. we get

$$l.p = l.u + l.\overline{1 - \frac{1}{r^t}} - l.\overline{r-1} \text{ theorem 1ſt,}$$

$$l.u = l.p + l.\overline{r-1} - l.\overline{1 - \frac{1}{r^t}} \text{ theorem 2d,}$$

$$\frac{l.u - l.\overline{u + p - pr}}{l.r} = t \text{ theorem 3d.}$$

191. Theorem 1st. Multiply the log. of the ratio by the time, and take the product from the log. of unity; then take one from the number which corresponds to the logarithmical difference; to the log. of which add the log. of the annuity, and from the sum take the log. of the ratio, less one: this difference gives the log. of the present worth.

Theorem 2d. To the log. of the present worth, add the log. of the ratio, less one, and reserve the sum; then multiply the log. of the ratio by the time, and take the product from the log. of unity; then take one from the number which corresponds to the logarithmical difference, and take the log. of this number from the log. reserved, and the difference is the log. of the annuity.

Theorem 3d. To the annuity add the present worth, and from the sum take the product of the present worth by the ratio, and take the log. of this quantity from the log. of the annuity, and divide the difference by the log. of the ratio, the quotient gives the time.

EXAMPLES.

192. Quest. 1st. What ready money will purchase an annuity of 100*l.* per annum, for 7 years, allowing 4*l.* per. cent compound interest?

Multiply the log. of $r = 1{\cdot}04 =$ ·0170333
by $t = 7$

·1192331

from the log of $1 =$ 0·0000000
take the log. of $r^t =$ 0·1992331

the difference is the log. of $\frac{1}{r^t} = {\cdot}7599 = \bar{1}{\cdot}8807669$

then to the log. of $u = 100 =$ 2·0000000
add the log. of $1 - \frac{1}{r^t} = {\cdot}2401 = \bar{1}{\cdot}3803922$

1·3803922
the sum is 1·3803922
From which take the log. of $r - 1 = {\cdot}04 = \bar{2}{\cdot}6020600$

the difference in the log. of $p = 610{\cdot}25 =$ 2·7783322

Queſt. 2d. What ready money will purchaſe an annuity of 100*l*. per annum, payable half yearly, to continue 7 years, allowing 4*l*. per cent per annum compound intereſt?

To the log of $u = 50l. =$ 1·6989700
add the log. of $1 - \frac{1}{r^t} = {\cdot}244 = \bar{1}{\cdot}3873898$

the ſum is 1·0863598

from which take the log. of $r - 1 = {\cdot}02 = \bar{2}{\cdot}3010300$

the difference is the log. of $p = 610l. =$ 2·7853898

Queſt.

Quest. 3d. What annuity, to continue 7 years at 4*l.* per cent per annum, may be purchased for 600*l.* 5*s.* at compound interest?

$$\text{To the log. of } p = 600\cdot25\,l. = 2\cdot7783322$$
$$\text{add the log. of } r - 1 = 04 = \bar{2}\cdot6020600$$

$$\text{the sum is } 1\cdot3803922$$

$$\text{from which take the log. of } 1 - \frac{1}{r^t} = \cdot2401 = \bar{1}\cdot3803922$$

$$\text{the difference is the log. of } u = 100\,l. = 2\cdot0000000$$

Quest. 4th. For what time will 600*l.* 5*s.* 0*d.* purchase an annuity of 100*l.* per annum, compound interest. at 4*l.* per cent per annum?

$$\text{From the log. of } u = 100 = 2\cdot$$
$$\text{take the log. of } u + p + pr = 76 = 1\cdot8808136$$

$$\cdot1191864$$

$$\text{Then } \frac{1191864}{\cdot0170333} = 7 \text{ years nearly.}$$

Quest. 5th. What ready money will purchase the reversion of a lease of 100*l.* per annum, to continue 7 years; but not to be entered upon till after the expiration of 9 years, allowing the purchaser 4*l.* per cent per annum, compound interest for his money?

The present value of the annuity to be entered upon immediately, is, per theorem 1st, = 600*l.* 5*s.* then, per theorem 2d, compound interest must be found: what principal, put to interest at 4*l.* per cent per annum, in 7 years will amount to 600*l.* 5*s.* which is accordingly found 421*l.* 19*s.* 6*d.* the present value of the lease.

Of

Of the PURCHASING *of* FREEHOLD *or* REAL ESTATES.

DEFINITION.

193. THE buying of eſtates is no more than the purchaſing an annuity to continue for ever, and therefore may be performed by the general theorem, art. 190, where $pr^t = \frac{ur^t - u}{r - 1}$. But t in this caſe being infinite, $ur^t - u$ will become ur^t only for a finite quantity, taken from an infinite one, the difference will be infinite; conſequently $pr^t = \frac{ur^t}{r-1}$ which being divided by r^t gives $p = \frac{u}{r - 1}$ theorem 1ſt,

$$p \times \overline{r - 1} = u \text{ theorem 2d,}$$

$$\frac{u}{p} + 1 = r \text{ theorem 3d.}$$

By theſe theorems, queſtions relative to the purchaſing freehold eſtates, are eaſily computed, and without the aſſiſtance of logarithms becauſe t is out of the equation.

EXAMPLES.

Queſt. 1ſt. Suppoſe a freehold eſtate of 25 *l.* per annum to be ſold, what is it worth, allowing $4\frac{1}{2}$ *l.* per cent. per annum to the purchaſer?

Per theorem 1. $\frac{25}{\cdot 045} = 555 \cdot \dot{5}\,l. = 555\,l.\ 11\,s.\ 1\frac{3}{4}\,d.$

Queſt. 2d. Admit a perſon lays out 1000 *l.* to purchaſe an eſtate, ſo as to have $3\frac{1}{2}$ *l.* per cent. per annum, compound intereſt what muſt the annual rent of the eſtate be?

Per

Per theorem 2. $1000 \times \cdot 035 = 35\,l.$ the anſwer.

Queſt. 3d. Suppoſe one gives 800 *l.* for an eſtate of 40 *l.* per annum, what rate per cent. compound intereſt, has the purchaſer for his money?

Per theorem 3. $\frac{40}{800} + 1 = 1 \cdot 05 = 5\,l.$ per cent. per annum.

Queſt. 4th. What ready money will purchaſe the reverſion of an eſtate of 100 *l.* per annum, not to be entered upon till 21 years are expired, allowing 5 *l.* per cent. per annum, compound intereſt, to the purchaſer.

First, $\frac{100}{\cdot 05} = 2000\,l.$ the preſent value of the eſtate.

Then per theorem 2. compound intereſt, we muſt find what principal, put to intereſt at 5 *l.* per cent. will amount to 2000 *l.* in 21 years.

Therefore from the log. of $a = 2000 = 3 \cdot 3010300$
Take the log. of $r^t = \cdot 4449753$

The dif. is the log. of $p = 717 \cdot 9 = 2 \cdot 8560547$

And hence we find, that an eſtate of 100 *l.* per annum, ſold according to the conditions of the queſtion, is worth no more than 719 *l.* 18 *s.*

Of ANNUITIES *on* LIVES, *&c.*

194. THE uncertainty of the lives of mankind renders the valuation of annuities on lives very precarious, and the purchaſing of them no more than a kind of gaming; and therefore the rules relative thereto are to be eſtimated from the ſame principles, viz. the laws

laws of chance. Now ſhould any one throw a die the chance that any ſide, ſuppoſe an ace come up, is $\frac{1}{6}$, for there are five chances to miſs, and only one for the contrary: Alſo ſhould two dice be thrown, the chance of their both coming up aces is $\frac{1}{6} \times \frac{1}{6}$ or $\frac{1}{36}$ and hence we derive the following general theorem, *That the chance of any event happening, is as the ſum of the chances which it has both to happen and to fail.*

Seeing the probability of the human life can be eſtimated from no other cauſes than the bills of mortality, we ſhall exhibit a Table calculated by Mr Simpſon for that purpoſe.

195. *A* TABLE *shewing the Probability of Life from the* BILLS *of* MORTALITY.

Ages	No of Persons living.	Ages	No of Persons living.	Ages	No of Persons living.
Born	1280	32	367	64	105
1	870	33	358	65	99
2	700	34	349	66	93
3	635	35	340	67	87
4	600	36	331	68	81
5	580	37	322	69	75
6	564	38	313	70	69
7	551	39	304	71	64
8	541	40	294	72	59
9	532	41	284	73	54
10	524	42	274	74	49
11	517	43	264	75	45
12	510	44	255	76	41
13	504	45	246	77	38
14	498	46	237	78	35
15	492	47	228	79	32
16	486	48	220	80	29
17	480	49	212	81	26
18	474	50	204	82	23
19	468	51	196	83	20
20	462	52	188	84	17
21	455	53	183	85	14
22	448	54	178	86	11
23	441	55	165	87	9
24	434	56	158	88	7
25	426	57	151	89	5
26	418	58	144	90	4
27	410	59	137	91	3
28	402	60	130	92	2
29	394	61	123	93	1
30	385	62	117	94	0
31	376	63	111		

196. Now if we look in the table againſt the age of 14, we ſhall find 498 perſons living out of the 1280; and at the age of 15 but 492, hence there has ſix died in the interval; and per theorem, (art. 194.) the chance that a perſon of 14 years of age has to live to 15, is as $492 + 6 : 492$, or as $\frac{492}{498}$.

197. Suppoſe a perſon 14 years of age have 1 *l.* to receive if he live to 15, quere the preſent value of his expectation, allowing 4 *l.* per cent. per annum, compound intereſt?

Solution.

The preſent value of 1 *l.* due 1 year hence, is found, per theorem 1ſt, art. 192, to be ·9675 *l.* and ſince his chance of living is $\frac{492}{498}$ therefore $\frac{492}{498} \times \cdot 9675 = \cdot 947 =$ 18 *s.* 11$\frac{1}{4}$*d.* the value of his expectation.

Should the value of 1 *l.* be required in like manner, for his chance of living 2 years, we find the preſent worth of one pound at the end of 1 year $= \cdot 9245$, and his chance to live $\frac{486}{498}$ therefore $\frac{486}{498} \times \cdot 9245 =$ 17 *s.* 11 *d.* and ſo we proceed to find the value of his expectation at the end of 3, 4, &c. years, even to the extremity of age, the ſum total of theſe expectations will be found to amount to 16 *l.* nearly, which is the value of 1 *l.* annuity on a life of 14 years of age.

After this manner the following table was computed, which exhibits the value of 1 *l* annuity, at 3, 4, and 5 *l.* per cent. compound intereſt, on any life from 14 to 75 years of age.

197. *A* TABLE *shewing the value of* 1 *l. annuity at* 3, 4, *and* 5 *l. per cent. on any life from* 14 *to* 75 *years of age.*

Age	Present worth of 1 *l.* annuity at 5 *l.* per cent.	Ditto at 4 *l.* per cent.	Ditto at 3 *l.* per cent.	Age	Present worth of 1 *l.* annuity at 5 *l.* per cent.	Ditto at 4 *l.* per cent.	Ditto at 3 *l.* per cent.
14	14·0	16·0	18·5	45	9·8	10·8	12·3
15	13·9	15·8	18·3	46	9·7	10·7	12·1
16	13·7	15·6	18·1	47	9·5	10·5	11·9
17	13·5	15·4	17·9	48	9·4	10·4	11·8
18	13·4	15·2	17·6	49	9·3	10·2	11·6
19	13·2	15·0	17·4	50	9·2	10·1	11·4
20	13·0	14·8	17·2	51	9·0	9·9	11·2
21	12·9	14·7	17·0	52	8·9	9·8	11·0
22	12·7	14·5	16·8	53	8·8	9·6	10·7
23	12·6	14·3	16·5	54	8·6	9·4	10·5
24	12·4	14·1	16·3	55	8·5	9·3	10·3
25	12·3	14·0	16·1	56	8·4	9·1	10·1
26	12·1	13·8	15·9	57	8·2	8·9	9·9
27	12·0	13·6	15·6	58	8·1	8·7	9·6
28	11·8	13·4	15·4	59	8·0	8·6	9·4
29	11·7	13·2	15·2	60	7·9	8·4	9·2
30	11·6	13·1	15·0	61	7·7	8·2	8.9
31	11·4	12·9	14·8	62	7·6	8·1	8·7
32	11·3	12·7	14 6	63	7·4	7·9	8·5
33	11·2	12·6	14·4	64	7·3	7·7	8·3
34	11·0	12·4	14·2	65	7·1	7·6	8·1
35	10·9	12·3	14·1	66	6·9	7·3	7·8
36	10·8	12·1	13·9	67	6·7	7·1	7·6
37	10·6	11·9	13·7	68	6·6	6·9	7·4
38	10·5	11·8	13·5	69	6.4	6·7	7·1
39	10·4	11·6	13·3	70	6·2	6·5	6·9
40	10·3	11·5	13·2	71	6·0	6·3	6·7
41	10·2	11.4	13·0	72	5·8	6·1	6·5
42	10·1	11·2	12·8	73	5·6	5·9	6·2
43	10·0	11·1	12·6	74	5·4	5·6	5·9
44	9·9	11·0	12·5	75	5·2	5·4	5·6

198. *To*

198. *To find the present value of an annuity according to the calculation of the preceding table.*

RULE. *Multiply the given annuity by the present value of* 1 *l. according to the age and rate per cent.*

Queſt. 1. What preſent money will purchaſe an annuity of 25 *l.* per annum on a life of 25 years of age, at 5 *l.* per cent. compound intereſt?

Againſt 25 years in the tables, and at 5 per cent. we have 12·3, and per rule, 12·3 × 25 = 307·5 = 307 *l.* 10 *s.* the preſent value of the given annuity.

Queſt. 2. Required the value of a 45 *l.* annuity on a life of 60 years of age, at 4 per cent. per annum?

Look in the table at 60 years, and 4 per cent. againſt which ſtands 8·4; therefore 45 × 8·4 = 378 *l.* the value of the annuity.

199. The calculation of annuities on two or three lives, or the longeſt liver, is of ſo intricate a nature, requiring ſuch an immenſe deal of trouble; and, upon the whole, neither ſo uſeful nor certain as thoſe on a ſingle life, that, I hope, (conſidering the intended brevity of the work) the omiſſion of them will be accounted no defect; ſhall therefore proceed to ſome examples which diſcover more of the utility of compound-intereſt annuities, *&c.*

Queſt. 1ſt, Two young Gentlemen, *A* and *B*, have each an eſtate of 1000 *l.* per annum, upon which they enter at the ſame time; *A* lives at the rate of 900 *l.* per annum, and *B* at 1100 *l.* per annum, for the firſt ten years. But *A* finding himſelf peſſeſſed of a conderable ſum of ready money, is determined to ſpend 1100 *l.* per annum. And on the other hand *B*, obſerving that he was involving himſelf in debt, reſolves to ſpend only 900 *l.* per annum; now allowing 5 *l.* per cent. per annum, in both caſes, how long will it be (reckoning

from their first entering on their estates) before the said estates are again of equal value.

SOLUTION.

By theorem 1st of annuities, we have the amount of 100 *l.* per annum, at 5 *l.* per cent. in ten years = 1258 *l.* which is the cash saved by *A*, and the debt of *B*.—— And the time that 100 *l.* will be in discharging a debt of 1258 *l.* is found, per theorem 3d, present worth of annuities, to be 21·85 years; and hence it appears, that from their first entrance upon the estates, until their being of equal value again, is 32 years nearly.

Quest. 2d, Suppose a person has 17 years to commence in an estate of 40 *l.* per annum, what is the reversion of such an estate for ever worth, after the expiration of the 17 years, the estate being sold at 30 years purchase *.

SOLUTION.

Per theorem 3d, of freehold estates, $\frac{\overline{30 \times 40 + 40}}{40 \times 40}$ $= 1\cdot0\dot{3}$ the ratio of the rate per cent. and per theorem 1st, present worth of annuities, we find the present value of an estate of 40 *l.* per annum, to continue 17 years, the ratio being $10\cdot\dot{3}$ = to 512 *l.* 15 *s.* 8 *d.* hence the reversion is worth 1200 *l.* − 512 *l.* 15 *s.* 8 *d.* = 687 *l.* 4 *s.* 4 *d.*

200. Or the value of the reversion may be readily found by taking theorem 1st, present value of annuities,

* This is the 8th question in the Gentlemen's Diary for the year 1770.

from

from theorem 1ſt of freehold eſtates, that is $\frac{u}{r-1}$ —

$$\frac{u-\frac{u}{r^t}}{r-1} = \frac{u}{r-1} - \frac{u}{r-1} + \frac{u}{r^t \times \overline{r-1}} = \frac{u}{r^t \times \overline{r-1}}$$

$= p$ the required theorem, which by the logarithms affords the following theorem for the reverſion of any eſtate, viz. $l.\ u. - l.\ r - 1 - l.\ r \times t = l.\ p$, which in words is as followeth. *From the logarithm of the annuity take the logarithm of the ratio leſs one, from which difference take the logarithm of the ratio multiplied by the time, and it will give the logarithm of the preſent worth of the reverſion.*

Queſt. 3d, A Gentleman has an offer of the reverſion of an eſtate of 100 *l.* per annum, after the expiration of 15 years, or the ſaid eſtate for 15 years to commence immediately. Whether is the better offer, allowing 5 *l.* per cent. per annum, compound intereſt, in both caſes?

SOLUTION.

The preſent value of the eſtate is, per theorem 1ſt, art. 193, found $= \frac{100}{\cdot 05} = 2000\ l.$

And the preſent worth of the reverſion is found per laſt theorem, thus:

From the log. $\frac{u}{r-1} = 2000 = 3\cdot 3010300$

Take the log. of $r = 1\cdot 05 = 0\cdot 211893 \times 15 = 0\cdot 3178395$

The difference is the log. of $p = 962\cdot 0342 = 29831905$

And from hence it is evident, that the term of 15 years is better than the reverſion for ever after, by 75 *l.* 18 *s.* 7½ *d.*

Queſt.

Queft. 4th, A Gentleman of 30 years of age, has an eftate of 40 *l.* per annum, for which he has two offers, viz. 80 *l.* per annum for life, or the prefent value of the eftate, allowing 5 per cent. compound intereft, in both cafes. Whether is the better offer?

		£.
The value of the annuity is	$11\cdot6 \times 80 =$	928
And the value of the eftate is	$\frac{40}{\cdot05} =$	800
Hence the annuity is better by		128

A COLLECTION *of* PRACTICAL COMPENDIUMS *in the Mercantile Way.*

1ft, BY having the price of 1 to find the price of 240.

RULE. *Call every penny (of the price) a pound, every farthing a crown,* &c *.

EXAMPLES.

To what comes a pound, or 240 pennyweights of filver, at $2\frac{3}{4}$ *d.* per pennyweight.

Per rule, the 2 pence is accounted 2 *l.* and the 3 farthings three crowns, therefore the anfwer is 2 *l.* 15 *s.* 0 *d.*

Required the value of a pack of ferges, containing 240 yards, at $14\frac{1}{2}$ *d.* per yard. Anf. 14 *l.* 10 *s.* 0 *d.*

* Becaufe 240 *d.* is 1 *l.* therefore every penny of the price muft be 1 *l.* &c.

Bought

Bought 2 *cwt.* or 240 *lb.* of Gloucester cheese, at $3\frac{7}{8}$ *d.* per *lb.* required the value? Ans. 3 *l.* 17 *s.* 6 *d.*

Sold 240 *lb.* of beef, at $3\frac{1}{4}$ *d.* required the price thereof? Ans. 3 *l.* 5 *s.* 0 *d.*

2. To find the value of the long hundred weight, by having the price of a pound given.

Rule. *For every penny per pound, account* 10 *s. per cwt. for every halfpenny a crown,* &c. *

Examples.

Required the value of a *cwt.* of fine flour, at $2\frac{1}{4}$ *d.* per *lb*?

Per rule, the 2 *d.* per *lb.* is 1 *l.* per *cwt.* and the farthing half a crown, therefore 1 *l.* 2 *s.* 6 *d.* is the answer.

To what comes $8\frac{3}{4}$ *cwt.* of second flour, at $1\frac{1}{2}$ *d.* per *lb.*

Now per rule, 1 *cwt.* is worth 15 *s.*, and 8 *cwt.* will come to 6 *l.* and the $\frac{3}{4}$ of 15 *s.* is 11 *s.* 3 *d.* therefore the answer is 6 *l.* 11 *s.* 3 *d.*

Required the value of 36 *cwt.* 3 *qr.* 10 *lb.* of Cheshire cheese, at $3\frac{3}{4}$ *d.*, per *lb*? Ans. 69 *l.* 0 *s.* $5\frac{1}{2}$ *d.*

To what comes 3 *cwt.* 3 *qr.* 20 *lb.* of flour at $2\frac{1}{8}$ *d.* per *lb*? Ans. 4 *l.* 2 *s.* 4 *d.*

What's the worth of 120 *lb.* of tea, at 7 *s.* 8 *d.* per *lb.*? Ans. 46 *l.* 0 *s.* 0 *d.*

* Seeing that the long *cwt.* is 120 *lb.* and 120 *d.* is 10 *s.* therefore the rule is evident.

To

To what comes 7 *cwt.* of Irish butter, at at $4\frac{3}{4}$ *d.* per *lb*? Ans. 16 *l.* 12 *s.* 6 *d.*

Suppose a person's wages be 1 *s.* 2 *d.* per day, what will he have to receive for 120 day's works? Ans. 7 *l.* 0 *s.* 0 *d.*

Admit Bacon be at $6\frac{1}{4}$*d.* per *lb.* what will 7 fliches, each 120 *lb.* come to? Ans. 21 *l.* 17 *s.* 6 d.

3d, Having the price of 1 *lb.* to find the price of a *cwt.* or 112 *lb.*

RULE. *Write down the price of a pound in farthings, doubling the units place for shillings, to which add* $\frac{1}{6}$ *of itself, which gives the answer* *.

EXAMPLES.

Required the value of a *cwt.* of sugar, at $4\frac{3}{4}$*d.* per *lb.*

	£.	s.	d.	
$4\frac{3}{4}$ *d.* = 19 *f.* = per rule, 1 *l.* 18 *s.* $\frac{1}{6}$	1	18	0	
		6	4	
£.	2	4	4	ans.

* Because 96 is $\frac{1}{10}$ of 960, the farthings in a pound Sterling, the farthings in the given price will be so many tenths of a pound Sterling, respecting the value of 96 *lb.* or so many two shillings as there are given farthings, therefore the value of 96 *lb.* is had per rule 3 in practice, and $\frac{1}{6}$ of 96 added to 96, gives 112, and hence the rule is deduced.

What's the price of 5 *cwt.* of iron at $3\frac{1}{4}$ *d.* per *lb*?

	£.	s.	d.
$\frac{1}{6}$	1	6	0
		4	4
	1	10	4
			5
	7	11	8 ans.

What's the value of $7\frac{1}{4}$ *cwt.* of hams, at $6\frac{1}{2}$ *d.* per *lb*? Ans. 23 *l.* 10 *s.* 2 *d.*

Required the value of 5 *cwt.* 2 *qr.* of raisins, at $5\frac{1}{4}$*d.* per *lb*? Ans. 13 *l.* 9 *s.* 6 *d.*

If 1 *lb.* of tobacco cost 9 *d.* what will 27 *cwt.* cost? Ans. 113 *l.* 8 *s.* 0 *d.*

4th. Having the price of an ounce, to find the price of 100 *lb.*

RULE. *Annex a cypher to the price of an ounce in farthings, $\frac{1}{6}$ whereof is the answer in pounds Sterling* *.

EXAMPLES.

To what comes 100 *lb.* of pepper, at $1\frac{1}{4}$ per ounce.

$\frac{1}{6}$ 50

£. 8 6 8 answer.

* A hundred pound is 1600 ounces; therefore if $\frac{1600}{960} = \frac{10}{6}$ of the price in farthings be taken, it will give the answer agreeable to the rule.

Required

Required the value of 112 *lb.* or a *cwt.* of tea, at $3\frac{3}{4}$ *d.* per ounce?

$$
\begin{array}{lrrl}
\frac{1}{6}) & 150 & & \\
\hline
\frac{1}{10}) & 25\,l. & & \text{the price of } 100\,lb. \\
\frac{1}{5}) & 2 & 10 & \text{the price of } 10 \\
 & & 10 & \text{the price of } 2 \\
\hline
£. & 28 & 0 & \text{the price of } 112\,lb.
\end{array}
$$

What's the value of 5 *cwt.* of cloves at $3\frac{1}{4}$ *d.* per ounce? Anſwer 121 *l.* 6 *s.* 8 *d.*

5th, Having the price of an ounce, to find the price of the long *cwt.*

Rule. *The price of an ounce in farthings doubled, gives the anſwer in pounds* *.

EXAMPLES.

Bought 120 *lb.* of coffee, at $3\frac{1}{4}$ *d.* per ounce, required the value thereof? $3\frac{1}{4}$ *d.* = 13 farthings, gives 26 *l.* anſwer.

Required the value of 120 *lb.* of ſnuff, at $1\frac{1}{4}$ *d.* per ounce. Anſ. 10 *l.* 0 *s.* 0 *d.*

To what comes 120 *lb.* of indigo at 16 *d.* per ounce? Anſ. 128 *l.*

6th, Given the price of the long *cwt.* to find the price of 1 *lb.*

Rule. *Account every pound Sterling of the price per cwt. equal to 2 d. per pound, every 10 s. equal to 1 d. and every half crown a farthing, &c.* †

* Now $\frac{120 \times 16}{960} = 2$, and hence it is that the farthings are doubled for pounds.

† This rule is the reverſe of the ſecond.

Ex-

If 1 *cwt.* or 120 *lb.* of cheese cost 1 *l.* 12 *s.* 6 *d.* what will 1 *lb.* cost?

Per rule, the 1 *l.* per *cwt.* is 2 *d.* per *lb.* 10 *s.* is 1 *d.* and 2 *s.* 6 *d.* a farthing: these summed give the answer $3\frac{1}{4}$ *d.*

Suppose 1 *cwt.* of flour cost 1 *l.* 7 *s.* 6 *d.* what is that per *lb*? Ans. $2\frac{3}{4}$ *d.*

Suppose 120 *lb.* of Irish butter cost 1 *l.* 17 *s.* 6 *d.* what is that per *lb*? Ans. $3\frac{3}{4}$ *d.*

If 120 *lb.* or 1 *cwt.* of wool cost 2 *l.* 2 *s.* 6 *d.* what is it per *lb*? Ans. $4\frac{1}{4}$ *d.*

7th, The price of the long *cwt.* being given, to find the price of an ounce.

RULE. *Take half of the price (of the given cwt.) in pounds; and the answer will be the price of an ounce in farthings**.

EXAMPLES.

Bought 120 *lb.* of coffee for 22 *l.* what did it cost per *lb*?

$$\frac{22}{2} = 11 \text{ farthings} = 2\tfrac{3}{4}\, d \text{ per ounce.}$$

Suppose 120 *lb.* of tea cost 24 *l.* 10 *s.* what will 1 ounce cost?

$$\frac{24\cdot 5}{2} = 12\cdot 25 = 3\tfrac{1}{16}\, d \text{ answer.}$$

Suppose 120 *lb.* of coffee cost 17 *l.* what will an ounce cost? Ans. $8\frac{1}{2}$ farthings $= 2\frac{1}{8}$ *d.*

Admit 120 *lb.* of pepper cost 12 *l.* 10 *s.* what's that per ounce? Ans. $1\frac{5}{8}$ *d.*

* This is the reverse of the 5th rule.

8th, The price of a *cwt.* or 112 *lb.* being given, to find the price of a pound.

RULE. *Take $\frac{1}{7}$ of the price of a cwt. from the said price, the difference in pounds with a cypher annexed will be farthings, and every shilling half a farthing* *.

EXAMPLES.

Suppose 112 *lb.* or a *cwt.* of tea cost 28 *l.* what is that per *lb?*

$$\begin{array}{rr} & £. \\ \frac{1}{7} & 28 \\ & 4 \\ \hline & 24 \end{array} = 240 \text{ farthings} = 5\,s. \text{ per } lb.$$

Sugar at 3 *l.* 0 *s.* 8 *d.* per *cwt.* what per *lb?*

$$\begin{array}{rrrr} & £. & s. & d. \\ \frac{1}{7} & 3 & 0 & 8 \\ & & 8 & 8 \\ \hline & 2 & 12 & 0 \end{array} = 26 \text{ farthings} = 6\tfrac{1}{2}\, d. \text{ per } lb.$$

Coffee at 35 *l.* per *cwt.* what per *lb?* Ans. 6 *s.* 3 *d.*

Hops at 10 *l.* per *cwt.* what per *lb?* Ans. 1 *s.* 9 $\frac{3}{8}$ *d.*

Loaf sugar at 10 *l.* 10 *s.* per *cwt.* what per *lb.* Ans. 1 *s.* 10 $\frac{7}{8}$ *d.*

Steel at 1 *l.* 15 *s.* per *cwt.* what per *lb.* Ans. 3 $\frac{7}{8}$ *d.*

Iron at 17 *s.* 6 *d.* per *cwt.* what per *lb.* Ans. 1 $\frac{7}{8}$ *d.*

9th, Having the value of a gallon, wine measure, to find the value of a tun.

RULE. *Call every shilling per gallon* 12 *guineas per tun, every penny a guinea,* &c.†

* This rule is the reverse of the 3d.

† The reason of this rule appears from 252 *d.* being 1 guinea, and 252 gallons a tun.

Ex-

EXAMPLES.

Wine at 5 *s.* 0 *d.* per gallon, what per tun?
$5 \times 12 = 60$ guineas $= 63$ *l.* per tun.

Brandy at 10 *s.* 6 *d* per gallon, how much per tun? $10{\cdot}5 \times 12 = 126$ guineas $= 132$ *l.* 6 *s.* per tun.

Wine at 5 *s.* 4 *d.* per gallon, what per tun? 5 *s.* 4 *d.* $= 5{\cdot}\dot{3}$ *s.* which multiplied by 12 gives $5{\cdot}\dot{3} \times 12 = 64$ guineas $= 67$ *l.* 4 *s.* the anſwer.

Oil at 3 *s.* 6 *d.* per gallon, what is it per tun? Anſ. 44 *l.* 2 *s.*

Geneva at 7 *s* 6 *d.* per gallon, what per tun? Anſ. 94 *l.* 10 *s.*

Rum at 8 *s.* per gallon, what per tun? Anſ. 100 *l.* 16 *s.*

10th, Having the price of a gallon, wine meaſure, to find the price of a hogſhead.

RULE. *Account every four-pence per gallon equal to a guinea per hogſhead,* &c. *or find the price of a tun per laſt rule, and half thereof will be the price of a pipe, or a fourth thereof the price of a hogſhead.*

EXAMPLES.

What will a pipe of wine come to at 5 *s.* 6 *d.* per gallon? Now $5{\cdot}\dot{5} \times 12 = 66$ guineas $= 69$ *l.* 6 *s.* per tun, the half of which is 34 *l.* 13 *s.* per pipe.

Canary wine at 6 *s.* 6 *d.* per gallon, what per Hogſhead? 6 *s.* 6 *d.* per gallon is 78 guineas per tun $= 81$ *l.* 18 *s.* the fourth of which is 20 *l.* 9 *s.* 6 *d.* per hogſhead.

Coniac brandy at 11 *s.* 6 *d.* per gallon, what will it come to per pipe? Anſ. 72 *l.* 9 *s.*

Jamaica rum at 6*s.* 8*d.* per gallon, what is that per pipe? Anſ. 42*l.* 0*s.* 0*d.*

Oil at 3*s.* 4*d.* per gallon, what per hogſhead? Anſ. 10*l.* 10*s.*

11th, By having the price of a tun, wine meaſure, to find the price of a gallon.

RULE. *Account every guinea per tun equal to a penny per gallon*, &c*.

EXAMPLES.

Admit a tun of canary coſt 64*l.* what is that per gallon? Now, 64*l.* is 60 guineas and 20*s.* and the 60 guineas per tun is 5*s.* per gallon, and 15*s.* 9*d.* per tun is 3*qr.* per gallon; therefore the anſwer is 5*s.* $0\frac{3}{4}$*d.*

Required the price of a gallon of that wine which coſt 63 guineas per tun? 63 guineas per tun is 63*d.* = 5*s.* 3*d.* per gallon.

Required the value of a gallon of Geneva at 70*l.* per tun? Anſ. 5*s.* $6\frac{1}{2}$*d.*

Brandy at 100*l.* per tun, what per gallon? Anſ. 7*s.* 11*d.*

Rum at 80*l.* per tun, what per gallon? Anſ. 6*s.* 4*s.*

Having the expences of one day, to find how much it is a year.

RULE. *Account every penny per day* 1*l.* 10*s.* 5*d.* *per year, or make the number of pence per day ſo many pounds, half pounds, and five-pences* †.

* This is the reverſe of the the 9th rule.

† For 240 days at a penny is 1*l.* 120 days is 10*s.* and 5 days 5*d.*

Ex-

EXAMPLES.

A perſon ſpends 9*d.* per day, what is that a year?

	£.		
Per rule 'twill be	9		
	4	10	
		3	9
	£. 4	13	9 answer.

A ſervant agrees with a gentleman for 1 *s.* 10*d.* per day, what is he to have yearly?

£.		
22		
11		
	9	2
£. 33	9	2 anſwer.

What is the amount of 19½ *d.* per day in a year?

	£.		
	19		
	9	10	
		7	11
and ½ of 1 *l.* 10 *s.* 5 *d.* =		15	2½
	£. 29	13	1½ anſwer.

If a perſon ſave 2 *s.* 4 *d.* per day, what does it come to in a year. Anſ. 42 *l.* 11 *s.* 8 *d.*

The expences of a day is 10¾ *d.* what will thoſe of a year be at that rate. Anſ. 16 *l.* 6 *s.* 11¾ *d.*

Suppoſe a perſon's daily income is 10*s.* 6¾*d.* what is it per annum. Anſ. 192*l.* 15*s.* 3¾*d.*

To multiply any integral number by 5.

RULE. *Annex a cypher to the given number, and take the half thereof, which is the anſwer**.

EXAMPLES.

Multiply 72965 by 5.

$$\begin{array}{r} \frac{1}{2})\ \underline{729650} \\ 364825 \text{ anſwer.} \end{array}$$

Multiply 72935967 by 5. Anſ. 364679885.

To multiply any integral quantity by 50, 500, &c.

RULE. *Annex to the multiplicand one cypher more than there is in the multiplier, and take half thereof, and it will be the anſwer.* As is evident from the laſt rule.

EXAMPLES.

Multiply 76965 by 500.

$$\begin{array}{r} \frac{1}{2})\ \underline{76965000} \\ \underline{38482500} \text{ anſwer.} \end{array}$$

Required the product of 765656 into 5000. Anſ. 3828280000.

* By annexing the cypher the given quantity is multiplied by 10, the half of which muſt neceſſarily be the product of 5.

To

To multiply any whole number by 25, 250, 2500, &c.

RULE. *Annex two cyphers to the multiplicand if you multiply by 25, three if by 250, four if by 2500, &c. a fourth of which will give the answer*.*

EXAMPLES.

Multiply 72958 by 25.

$\frac{1}{4}$) 7295800

1823950 answer.

Required the product of 97507 by 2500. Ans. 243767500.

Required the product of 7965 by 25000. Ans. 199125000.

To find the product of any integral quantity by 75, 750, &c.

RULE. *For 75 annex two cyphers to the multiplicand, three for 750, &c. and take the half thereof, and again the half of that quotient, the sum of these two is equal the required product†.*

* Any quantity to which two cyphers are annexed is multiplied by 100, a fourth of which is 25, and in like manner any number multiplied by a 1000 will have 3 cyphers annexed, $\frac{1}{4}$ of which is 250, &c.

† If it be considered that $\frac{3}{4}$ of 100 is 75, the reason of this rule will appear as evident as the former.

EXAMPLES.

Multiply 176536 by 75.

½	17653600
½	8826800
	4413400
	13240200 answer.

Multiply 37656736 by 7500. Anſ. 282425520000.

Many other contractions, both in practical arithmetic and reſpecting the properties of numbers, might be added, but theſe may ſuffice as a ſpur to excite the ingenious learner in his purſuit of others equally as intereſting; and he may be aſſured, that the diſcovering them himſelf will make a more laſting impreſſion, and tend more to the advancement of his knowledge in figures than the precepts of the moſt ſkilful teacher. I ſhall therefore conclude with one remarkable property of numbers, viz. *That any number, of the ſame digits, ſquared, (or multiplied by themſelves) give a ſeries of numbers in arithmetical progreſſion, of which the middle term is the greateſt, and that the terms on each ſide decreaſe equally; the ſquare of the digit you multiply by being the common ratio or difference.*

EXAMPLES.

```
1111            2 2 2 2               3 3 3
1111            2 2 2 2               3 3 3
----            -------               -----
1111            4 4 4 4               9 9 9
 1111             4 4 4 4               9 9 9
  1111              4 4 4 4               9 9 9
   1111               4 4 4 4         ------------
-------         -------------------   9, 18, 27. 18, 9
1234321         4, 8, 12, 16, 12, 8, 4
```

By

By inſpecting the examples we ſhall find that the number of terms in the product (as there obtained) will always be one leſs than double the digits you would have ſquared. Now the firſt term is the ſquare of the given digit, and each of the following terms increaſe to the middlemoſt by the addition of the ſquare of the ſaid given digit; and from the greateſt term decreaſe in the ſame order. Thus, if the ſquare of 4444 is required, the terms will be 16 32 48 64 48 32 16, and the actual product of the ſaid digits will be obtained by the following device, viz. *putting the tens place of the right hand term (in the above ſeries) under the units place of the next term,* &c. *and taking the ſum thus*:

6 2 8 4 8 2 6 the digits in the units place.

1 3 4 6 4 3 1 the digits in the tens place.

1 9 7 4 9 1 3 6 the ſum = 4444 ſquared.

Again, if the ſquare of 66666 be required, the arithmetical ſeries will be 36 72 108 144 180 144 108 72 36; and the digits which compoſe the ſeries and of which we want the ſum, as below:

6 2 8 4 0 4 8 2 6

1 1 1 1 1

3 7 0 4 8 4 0 7 3

4 4 4 4 3 5 5 5 5 6 = the ſquare of 66666.

This property of numbers is to me entirely new, having never ſeen any thing in print which explained the nature thereof, though I had the hint from Mr Hutton's Mathematical Miſcellany, queſtion 27, where *Arithmeticus* ſhews it to be the property of the digit 1. I preſume the knowledge of this particular may tend to the diſcovery of ſome intereſting and uſeful properties of numbers of which we are as yet ignorant; for I am perſuaded, that we are far from being acquainted with all the properties of theſe digits, as is fully teſtified by the improvements made by the mathematicians of our own times.

BILLS of PARCELS AND BOOK DEBTS.

Mr James Goodfellow

BOUGHT *of Geo. Mercer*

Carlisle, 17th June, 1772.

	£.	s.	d.
SIX $\frac{3}{4}$ *yards* of double mill'd *plains* at 6*s.* 9*d.* per yard ——— ———	2	5	$6\frac{3}{4}$
7 ——— ——— of *shalloon*, at 1*s.* 6*d.*		10	6
2 *Shammy skins*, at 14*d.* ——— ——— ———		2	4
2 Dozen of *large plated buttons*, at 4*s.* 8*d.*		9	4
3 ——— of *small ditto*, at 2*s.* 6*d.* ———		7	6
$1\frac{1}{2}$ *Yards* of *flannel*, at 1*s.* 3*d.* ———		1	$10\frac{1}{2}$
$1\frac{5}{8}$ *Yards* of *Buckram*, at 1*s.* 4*s.* ——— —		2	
Thread 4*d.* *Twist* 8*d.* *Silk* 6*d.* *Canvas* 1*s.* 4*d.* and *Stay-tape* 3*d.*		3	1
	£. 4	2	$1\frac{1}{4}$

Received (the same date) the contents of the above in full, per GEO. MERCER.

Mr Miles Innkeeper

BOUGHT *of James Vintner*

	£.	s.	d.
63 Gallons of Port wine, at 5*s.* 9*d.* per gal.			
27 ——— ——— claret, at 7*s.* 6*d.* ——— —			
126 ——— Lisbon white, at 4*s.* 9*d.*			
$29\frac{1}{2}$ ——— sherry, at 6*s.* 6*d.*			
15 ——— Coniac brandy, at 10*s.* 6*d.*			
25 ——— rum, at 7*s.* 9*d.*			
10 ——— best Holland geneva, at 7*s.* 10*d.*			
$9\frac{1}{2}$ ——— common *ditto*, at 5*s.* 8*d.*			
	£. 91	18	5

Lan-

Lancaſter, 20th June, 1772.

Dear Sir,

Agreeable to your order I have ſent per Peter Careful, carrier, the above; all of which you'll find very good, and as low charged as poſſible. I purpoſe being in your town about the latter end of September, when ſhall expect (according to the deſert of this) your further orders. I am,

With the utmoſt ſincerity,
Your obliged humble ſervant,
JOHN TRUSTY.

Meſſrs Fell and Irwin

BOUGHT *of Tim. Brown,*

Carliſle, 28th June, 1772.

	Cwt.	qr.	lb.	£.	s.	d.
12 Cheſhire cheeſes, weight at 1*l.* 16*s.* 6*d.* per *cwt.*	4	2	14			
24 Lancaſter ditto at 1*l.* 14*s.*	5	1	7			
7 Gloceſter ditto at 2*l.* 2*s.*	0	3	4			
27 Fleaches of bacon at 2*l.* 11*s.* 4*d.*	12	3	7			
18 Middles ——— —— at 2*l.* 11*s.* 4*d.*	5	1	14			
44 Hams ——— ——— — at 2*l.* 16*s.*	8	0	7			
52 Firkins of butter, at 1*l.* 11*s.* 6*d.*						
6 Seconds ditto, at 1*l.* 7*s* 6*d.*						
				£.		

GENT.

The above I have forwarded to Newcaſtle, to the care of Mr Lawton, to ſhip for your account; ſhould have been glad if the prices had been lower, but that poſſibly could not be without loſs, we paid ſo extravagantly dear for green hams and bacon; theſe however, I hope, will prove very good; I was careful as poſſible in examining them. I am,

Your moſt obedient humble ſervant,
TIM. BROWN.

Mr

Mr John Simpson

BOUGHT *of Christopher Cornfactor*

Scaleby, 30th of June, 1772.

	£.	s.	d.
350 Bushels of barley, at 2 *s.* 8 *d.* per bush.			
1320 ——— of wheat 5 *s.* 6 *d.* —			
200 ——— of skill'd barley, at 5 *s.* 2 *d.*			
1760 ——— of oats, at 2 *s.* 2 *d.* ——			
156 ——— of Malt, at 3 *s.* —— ——			
6 bags of hops, at 3 *l.* 10 *s.* per bag			
$3\frac{1}{2}$ quarters of beans, at 7 *l.* 15 *s.* p. *qr.*			
4 ——— of peas, at 5 *l.* —— —			
£.			

Mrs Mary Blyth

BOUGHT *of Mr John Dixon, Hosier,*

Kendal, 23d June, 1772.

	£.	s.	d.
5 Dozen worsted stockings, at 36 *s.* 6 *d.* p. doz.			
$4\frac{1}{2}$ ——— ribb'd ditto, at 38 *s.* 9 *d.*			
4 ——— cotton ditto, at 42 *s.* 8 *d.*			
$5\frac{1}{4}$ ——— plain ditto, at 40 *s.* 6 *d.*			
4 ——— silk and cotton ditto, at 3 *l.* 10 *s.*			
$1\frac{1}{2}$ ——— silk ditto, at 6 *l.* 15 *s.* 6 *d.*			
15 Pair of women's silk gloves, 3 *s.* 9 *d.* p. pair			
7 ——— Mitts, at 3 *s.* 6 *d.*			
6 Breeches pieces, silk, at 14 *s.* 6 *d.* p. Piece			
28 Yards of flannel, at 1 *s.* $4\frac{1}{2}$ *d.* per yd.			
27 Hanks of worsted wt. 30 *lb.* at $2\frac{1}{8}$ *d.* per ounce			
£.	87	8	$2\frac{1}{4}$

F I N I S.